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Life insurance policy termination and survivorship[☆]

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HIGHLIGHTS

- We model the mortality of terminated contracts to understand lapses and survival.
- We find statistical evidence of mortality selection based on insurance company data.
- These results indicate a 50–50 chance of a positive loss when a policy terminates.

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ABSTRACT

There has been some work, e.g. Carriere (1998), Valdez (2000b), and Valdez (2001), leading to the development of statistical models in understanding the mortality pattern of terminated policies. However, there is a scant literature on the empirical evidence of the true nature of the relationship between survivorship and persistency in life insurance. When a life insurance contract terminates due to voluntary non-payment of premiums, there is a possible hidden cost resulting from mortality antiselection. This refers to the tendency of policyholders who are generally healthy to select against the insurance company by voluntarily terminating their policies. In this article, we explore the empirical results of the survival pattern of terminated policies, using a follow-up study of the mortality of those policies that terminated from a portfolio of life insurance contracts. The data has been obtained from a major insurer which traced the mortality of their policies withdrawn, for purposes of understanding the mortality antiselection, by obtaining their dates of death from the Social Security Administration office. Using a representative sample of this follow-up data, we modeled the time until a policy lapses and its subsequent mortality pattern. We find some evidence of mortality selection and we consequentially examined the financial cost of policy termination.

1. Introduction and literature

A life insurance contract is one very unique and complicated consumer product to put a price tag on. By the very nature of the product, its price has to be determined even before its actual underlying cost can be accurately assessed. Although actuaries responsibly predict the cost, it certainly can take several years before its ultimate price can indeed be determined. When determining a premium to assess for the product, the actuary makes its best estimate according to factors that directly affect the cost of the product including, but not limited to, the level and pattern of benefits, policy features and guarantees, expected returns on investments, and expenses. Some assumptions about mortality pattern and policy termination, together with the uncertainties and possible resulting variabilities associated with them, will additionally be required in the calculation of a premium. See Atkinson and Dallas (2000).

However, additional to this already complicated factors and assumptions is the consideration of the effect of policyholder behavior. One school of thought asserts that buyers of insurance products behave or react differently in the presence of insurance coverage. See Kunreuther et al. (2013). Whether consumers act rational or not in the presence of insurance, it is a widely accepted perception that there is an asymmetry of information in the insurance market. Under several circumstances, the insurer does not often have

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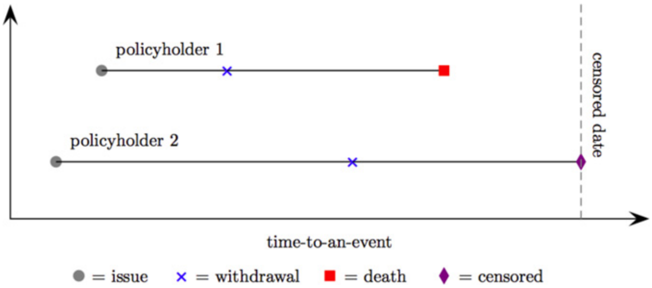


Fig. 1. Illustrative diagram of the observed times until withdrawal and death.

all the available information to accurately underwrite the level of risks of potential policyholders. Nevertheless, the insurer relies on the pooling of risks based on the assumption that there will be a large enough number of homogeneous risks so that the expected aggregate cost of insurance can be determined with reduced variability. The presence of asymmetric information has the potential to distort this homogeneity within the group and the resulting tendency is the pooling of more risks considered "worse" than the average risk. See, for example, the illustration of Bluhm (1992) in the case of health insurance policies. Several undesirable consequences of policyholder behavior may be mitigated so long as the insurer practices prudent risk management.

In this paper, we focus on policy termination together with understanding the survivorship pattern resulting from terminated policies. Our observable data is an extract from a real life data of a portfolio of terminated life insurance policies from an undisclosed insurance carrier which tracked the mortality dates of these policies from the US Social Security Administration office. The primary purpose of obtaining such information is to first understand the relationship between policy termination and mortality, and later, more importantly, to assess the financial implication of this relationship in the design, pricing and risk management of insurance products. These terminated policies are as of a fixed date, hereby undisclosed to preserve some level of confidentiality. The recorded death date information is also as of this same fixed date which is then considered the censoring date used in our model calibration. Our data file also recorded period around 1920s as the year with the earliest policy issue date in the portfolio. On the aggregate, we have observations totaling to 65,435 terminated single life policies, discarding joint life policies for purposes of our analysis. This set of observations that we use for model calibration in this paper is only a random sub-sample from the insurer's portfolio used in their analysis.

The type of observations in our empirical data is vividly illustrated in Fig. 1. According to this figure, we observe two distinctly classified policyholders, herewith labeled policyholders 1 and 2, where in both cases, we observe the times when each withdraw their policy out of the insurance company. Policyholder 1 dies before the end of the observation period and therefore we can observe its time from withdrawal until time of death. On the other hand, policyholder 2 is still alive at the end of the observation period and is therefore clearly considered a right-censored observation.

Given a policy is issued at a fixed and known age, denote this by z, we are interested in estimating the probability distribution of the time-until-withdrawal and the time-until-death from issue. Denote these times to events, respectively, by the random variates T_w and T_d, and define the difference as $T_{wd} = T_d - T_w$. Our data file allows us to observe T_w and the conditional random variate $T_{wd}|T_w$, or effectively $T_d|T_w$ since $(T_{wd}|T_w) = (T_d|T_w) - T_w$. For notation purposes, we can express $T^*_{wd} = T_{wd}|T_w$ and $T_{d|w} = T_d|T_w$. Notice that because not all policies were followed up until their times of death, censoring is therefore present and the observable T_d is therefore calculated as of the censoring date, as previously explained, and a censoring variable is recorded for each of the policies in the portfolio. These are indeed called right-censored observations which are typical in mortality studies. See, for example, Elandt-Johnson and Johnson (1980). Of our entire observations, we found that we have a total of 61,889 right-censored observations. Slightly over 5% of our observations are deaths, something not atypical of mortality follow-up studies.

Understanding, recognizing and measuring the relationship between policy termination and survivorship in a life insurance portfolio can be of considerable importance to actuaries with pricing, reserving and risk management responsibilities. This interconnection between termination and survivorship is something called in the insurance literature as mortality selection. This selection roughly produces an unbalanced mixture of low and high mortality risks which could have a spiraling effect on the insurer's portfolio of policies. Policyholders who terminate their policies are believed to have better mortality risks than those who remain; these policyholders are able to seek coverage elsewhere at possibly better premium rates. On the other hand, those who remain will have the effect of a worse mortality pattern than would have been originally anticipated creating a circumvolved early death claims. See the monograph published by Munich RE on how life insurance companies may combat such selection; see Donnelly (2011).

The extent of the probable damage caused by mortality selection will vary according to the nature and type of the life insurance product. For instance, in a traditional whole life insurance, the consequences could range from low to unmanageably high. Possible factors that contribute to this extent include the permanent long term nature of the commitment where the periodic payment of the premium is fixed at issue; policy alteration may be possible but within contractual constraints. Furthermore, this relationship may be more important in the life settlement business which as of late has turned into a multi-billion dollar industry. With life settlements, life insurance contract holders have the option of selling their policies to third party investors in the capital market, in lieu of terminating their policies. Such an arrangement has the potential financial attractiveness both to the policyholder and the investor. See, for example, Doherty et al. (2002) and Vadiveloo (2005). In this type of market, the investors would be interested in the survivorship patterns of these potentially terminating policies. The typical pricing approach for life settlement policies is to assume that the insured lives will have impaired mortality. This is a contrast to the mortality selection you would expect from a portfolio of ordinary life insurance policies. Finally, the recognition of the relationship between termination and survivorship is probably more importantly pronounced in lapse-supported products, something more common in Canada introduced in the early 1980s. See Tullis and Polkinghorn (1992). Pricing for lapse-supported products is very sensitive to the assumption of the proportions of policyholders terminating their policies; these are policies that provide long term commitments without the attraction of a cash surrender value at policy termination.

It is an unfortunate situation that there is a scant research work published in the literature about the true nature of the relationship between policy termination and survivorship. This is not to say that researchers and practitioners do not recognize its relevance, indeed far from it. In practice, the common method is to select average mortality and lapse rates, on the "aggregate" as Jones (1998) points out, applied to a class of contracts and policyholders. The aggregation, of course to the extent it is measurable, may vary according to the heterogeneous characteristics of the policyholders within a portfolio. Mortality impairment in subsequent periods is then reflected through the use of excess lapse rates for renewed policies. See also Dukes and MacDonald (1980). This approach is quite commonplace and recommended in reserving guidelines for annually renewable term policies, where the impact of selective lapsation is believed to be more pronounced; see for example, NAIC Actuarial Guideline XXX applicable for term insurance products.

In the research front, there has been some advances to suggest theoretical approaches in analyzing the relationship between mortality and lapsation. Jones (1998) recommended the use of "frailty" concept to account for the heterogeneity resulting from this relationship within a multi-state framework. Sigalotti (1988) suggested a Bayesian framework to possibly account for the "correlation between mortality and withdrawal". Carriere (1998) analyzed the impact of withdrawal benefits, in the presence of mortality anti-selection, within a double decrement framework. He demonstrated that when mortality and withdrawal decrements are independent, the withdrawal benefit should be the policy reserve, but should be much smaller in the presence of anti-selection. A nice follow up to the work of Carriere (1998), Valdez (2001) suggested the use of a "copula" framework to examine and quantify the effect of anti-selection. A more recent interesting work is that by DeGiovanni (2010) which capitalizes on the use of financial options theory to model policyholder behavior in much the same fashion as exercising American options.

Regrettably, there is little or no research work in circulation to calibrate these models in order to statistically support empirical evidence. There are several reasons for this. One is understandably the difficulty of academic researchers to obtain real life data to calibrate possibly hypothetical models. In effect, as a creative proxy, some obtain readily available published data to tweak the research issue originally raised and to focus on understanding relationships based on the available information. To illustrate, for example, the work of Tsai et al. (2002) used data that comes in the form of a time series of various macro-economic variables to incorporate the empirical relation between lapse and interest rates, and to examine the impact of this relationship on policy reserves. In addition, by the very nature of the problem of understanding the relationship between policy lapses and deaths, even observable data from the insurance company under several circumstances can make the calibration really difficult. The intuition here is that no insurance company is able to track the mortality pattern of terminated contracts. Such problem is often referred to in statistics as "model identifiability" but there are ways to circumvent around this issue including, but not limited to, the tightening and sharpening of the model specification.

Our approach, in this paper, can be considered more novel than several of these published research works. The nature of our observed data allows us to calibrate models at a micro-level, meaning observations are at the policyholder level. Many actuarial models are developed based on grouped data, but there is more information obtained at a micro-level allowing us to better reflect reality. With micro-level data becoming more available especially to practitioners, there is an increasing trend of developing micro-econometric models, a term used for example by Gourieroux and Jasiak (2007). Next, we use a general class of duration models to specify the parametric distribution of the time-until-withdrawal, T_w, from issue date. This class falls within the general framework of regression models for which the distribution of the error component can in some sense be arbitrarily specified. Not only is this class of models very tractable, but they apparently allows us to incorporate covariate terms within which in our context are policyholder characteristics such as gender, issue age, and product type. Duration models are commonly used in the field of econometrics, e.g. Gourieroux and Jasiak (2007) and Lancaster (1990). We find that for our data, the most suitable duration model for T_w is one where the error component follows a standard Gamma distribution, and we shall observe later that this indeed results in a Generalized Gamma distribution specification for T_w. Other parametric error distributions were also examined but these models provided weak statistical support to the data.

In examining parametric models for the time-until-death, conditionally on observing the time-until-withdrawal, $T_{d|w}$, we find that the best approach is to directly specify the distribution model for this random variate. According to our analysis, we find that the Gompertz model, something long well-known to actuaries, fits our survival data very well. This specification also allows us to incorporate issue age directly to the model, as well as gender through the parameters. It is widely known that survival deteriorates with age and that females live longer; both of these preconceived notions are reflected in our estimated models. We also investigated other observed characteristics, e.g. plan types and face amounts, that may explain the heterogeneous nature of the survival pattern among our observations; we found that many of these are not statistically significant. For comparative purposes and in probing the robustness of our models, we considered several other classes of survival models; not to overwhelm the reader, we only report in this article the second best model which is the Weibull.

Finally, once we have calibrated the parameters in the respective models for T_w and $T_{d|w}$, straightforward statistical results allow us to specify the unconditional distribution for the time-until-death, T_d. Consequently, this offers us the ability to make a comparison of survival patterns between the conditional random variate $T_{d|w}$ and the unconditional random variate T_d to provide us an understanding of possible evidence of mortality anti-selection. Using the definition of mortality anti-selection used in Valdez (2001) and in Carriere (1998), we are able to assess for the presence of this anti-selection. Finally, by examining a hypothetical insurance portfolio, we additionally assess the financial impact of this anti-selection. See also Valdez (2000b).

The structure of this paper is as follows. Section 2 introduces the class of duration models with direct specification of the error distribution within a regression framework. Here, we give as examples large families of regression models including the ordinary regression model where the error component is assumed to have Normal distribution. This section also discusses the Gompertz, together with the Weibull model, for modeling the age at death random variate. Section 3 provides a discussion of the characteristics of the data, together with results of preliminary statistical analysis which traversed us to our model choices. Section 4 provides the results and analysis of calibrating the models. Section 5 explains how these models are used to assess the presence of mortality antiselection together with a consideration of quantifying to understand the financial impact of these mortality antiselection. We conclude in Section 6.

2. Parametric models

2.1. A class of duration models for time-until-withdrawal

Consider the time-until-withdrawal random variate, T_w, referring to the duration that the policyholder lapses from date of issue, which clearly has a range of non-negative values. We shall denote its survival, distribution and density functions by S_w, F_w and f_w, respectively. These functions are related, for example, as follows:

$$S_w(t) = P(T_w > t) = 1 - F_w(t) = \int_t^\infty f_w(s)ds.$$

Suppose that we can write T_w as

$$T_w = \exp(\mu)T_0^\sigma, \tag{1}$$

for some non-negative random variate T_0. By re-writing (1) through the log-transformation

$$\log(T_w) = \mu + \sigma \log(T_0) = \mu + \sigma \Lambda, \tag{2}$$

where $\Lambda = \log(T_0)$, we observe that μ is a location parameter and σ is a scale parameter with the restriction that $\sigma \neq 0$ in order to avoid a degenerate distribution for T_w. Because we can write the survival function of T_w as

$$S_w(t) = \begin{cases} S_\Lambda\left(\dfrac{\log(t)-\mu}{\sigma}\right), & \text{for } \sigma > 0 \\ 1 - S_\Lambda\left(\dfrac{\log(t)-\mu}{\sigma}\right), & \text{for } \sigma < 0 \end{cases} \tag{3}$$

where S_Λ denotes the survival function of Λ, the distribution of T_w indeed belongs to a log-location-scale family of distributions.

Covariates can be introduced through the location parameter μ. Suppose $\mathbf{x}$ is a vector of covariates, such as policyholder characteristics, and β, the corresponding vector of linear coefficients. Then we can simply replace $\mu = \mathbf{x}'\beta$. For example, (1) becomes

$$T_w = \exp(\mathbf{x}'\beta) T_0^\sigma, \tag{4}$$

and (2) becomes

$$\log(T_w) = \mathbf{x}'\beta + \sigma \log(T_0) = \mathbf{x}'\beta + \sigma\Lambda, \tag{5}$$

which generalizes the familiar ordinary regression model where the error component has a Normal distribution. The specification in (4) is also a special case of the Accelerated Failure Time (AFT) model commonly studied in survival models. See Elandt-Johnson and Johnson (1980).

It is also straightforward to find the distribution of T_w in terms of the distribution of T_0. The survival distribution function of T_w can be expressed as

$$S_w(t) = S_0\left((e^{-\mu}t)^{1/\sigma}\right) \tag{6}$$

and its density as

$$f_w(t) = \frac{1}{|\sigma|t}(e^{-\mu}t)^{1/\sigma} f_0\left((e^{-\mu}t)^{1/\sigma}\right), \tag{7}$$

where S_0 and f_0 are respectively the survival and density functions of T_0. Within this class of models, it is oftentimes more straightforward to specify the distribution of T_0 rather than of its logarithm.

Example 1 (*Log-Normal Distribution*)**.** As an illustration, in the case where T_0 has a log-normal distribution with parameters 0 and 1, it can be shown that

$$f_w(t) = \frac{1}{\sqrt{2\pi}\sigma t} \exp\left[-\frac{1}{2}\left(\frac{\log(t)-\mu}{\sigma}\right)^2\right] \tag{8}$$

which also gives T_w a log-normal distribution with parameters μ and σ, where $\sigma > 0$. This distribution is well-studied both in finance and actuarial science.

Example 2 (*Generalized Gamma Distribution*)**.** Here, we suppose that T_0 has a standard Gamma distribution, i.e. one where the scale parameter is 1 but with a shape parameter m so that its density can be expressed as

$$f_0(y) = \frac{1}{\Gamma(m)} y^{m-1} e^{-y}.$$

It can be shown that

$$f_w(t) = \frac{1}{|\sigma|t} \frac{1}{\Gamma(m)} (e^{-\mu}t)^{m/\sigma} \exp\left[-(e^{-\mu}t)^{1/\sigma}\right]. \tag{9}$$

This gives a large class of distributions called the *Generalized Gamma* with parameter vector (μ, σ, m). For a member of this class, we shall write it as $T_w \sim \mathrm{GG}(\mu, \sigma, m)$. This family of distributions which is often attributed to Stacy (1962), includes as special cases the Gamma, Exponential, Log-Normal and Weibull distributions. Despite its flexibility, this family of distribution is less-studied in finance and actuarial science.

Example 3 (*GB2 Distribution*)**.** Here, we suppose that T_0 has a Beta of the second kind (B2) distribution whose density is expressed as

$$f_0(y) = \frac{1}{B(\gamma_1, \gamma_2)} \frac{y^{\gamma_1 - 1}}{(1+y)^{\gamma_1+\gamma_2}}.$$

This type of distribution is sometimes called the standard form of a Pearson Type VI distribution, see Johnson et al. (1995). It can be shown that

$$f_w(t) = \frac{1}{|\sigma|t} \frac{1}{B(\gamma_1, \gamma_2)} \frac{(e^{-\mu}t)^{\gamma_1/\sigma}}{\left[1 + (e^{-\mu}t)^{1/\sigma}\right]^{\gamma_1+\gamma_2}}. \tag{10}$$

This gives a large class of distributions called the GB2, or Generalized Beta of the second kind, with parameter vector $(\mu, \sigma, \gamma_1, \gamma_2)$. For a member of this class, we shall write it as $T_w \sim \mathrm{GB2}(\mu, \sigma, \gamma_1, \gamma_2)$. This family of distributions was first studied by McDonald (1984) and has been applied in modeling insurance claims, e.g. Cummins et al. (1990). As pointed out by Sun et al. (2008), it is well suited for fitting heavy-tailed data.

2.2. Survival models for the age at death random variable

Denote the (fixed) issue age by z and let X_d be the age at death random variable so that

$$X_d | z = z + T_w + (T_d - T_w) = z + T_w + T_{wd},$$

provided $T_{wd} > 0$. Thus, if T_w is known, then

$$(X_d | z, T_w = t_w) = z + t_w + T_{wd}.$$

Thus, it becomes clear that

$$\begin{aligned} P(T_{wd} > t_{wd} | z, T_w = t_w) &= P(T_d > T_w + t_{wd} | z, T_w = t_w) \\ &= \frac{P(X_d > z + t_w + t_{wd})}{P(X_d > z + t_w)} \\ &= \frac{S_d(z + t_w + t_{wd})}{S_d(z + t_w)}. \end{aligned} \tag{11}$$

Here, S_d refers to the corresponding survival distribution function of the age at death, X_d, random variable. That is, $S_d(x) = P(X_d > x)$. Driven by the observable data, we need to specify the distribution model for T_{wd}, given the issue age z and the time-until-withdrawal t_w. According to (11), this is equivalent to specifying the distribution model for the age at death random variable X_d. While we have examined several survival models for the age at death, the two models, both of which are commonly known to actuaries, are the Gompertz and the Weibull distributions.

Example 1 (*Gompertz Distribution*)**.** For the Gompertz distribution, we write its survival function in the form

$$S_d(x) = \exp\left[e^{-m^*/\sigma^*}\left(1 - e^{x/\sigma^*}\right)\right], \tag{12}$$

where $m^* > 0$ is the mode and $\sigma^* > 0$ is a dispersion measure about this mode of the distribution. This reparameterization has been suggested by Carriere (1992) and is being used here both for ease of parameter interpretation and estimation. By re-expressing the parameters with

$$B = \frac{1}{\sigma^*}\exp(-m^*/\sigma^*) \quad \text{and} \quad c = \exp(1/\sigma^*), \tag{13}$$

it leads us to the hazard function

$$\mu_x = \frac{f_d(x)}{S_d(x)} = Bc^x.$$

This simple expression is quite familiar to actuaries and has been well-studied in the actuarial literature. See, for example, Gompertz

Table 1
Some other parametric survival distribution models.

Distribution	Survival function $S_d(x)$	Force of mortality μ_x
Exponential	$\exp(-\mu x)$	μ
Inverse-Gompertz	$\frac{1-\exp\left[-e^{(x-m)/\sigma}\right]}{1-\exp(-e^{m/\sigma})}$	$\frac{1}{\sigma}\exp\left(\frac{x-m}{\sigma}\right)$
Inverse-Weibull	$1-\exp\left[-(x/m)^{-m/\sigma}\right]$	$\frac{(1/\sigma)(x-m)^{-(m/\sigma)-1}}{\exp\left[(x/m)^{-m/\sigma}\right]-1}$

(1825), Carriere (1992), Frees et al. (1996), and Bowers et al. (1986). There is an additional interesting property of the Gompertz model that may allow us to take advantage of especially when graphically examining candidate models for survival data. Consider the probability that an individual, now age x, will survive another year:

$$p_x = \frac{S_d(x+1)}{S_d(x)} = \exp\left[e^{(x-m^*)/\sigma^*}(1-e^{1/\sigma^*})\right]. \tag{14}$$

It becomes rather straightforward to see that $\log(\log(p_x))$ is linear in age x, that is,

$$\log(\log(p_x)) = a + bx, \tag{15}$$

where clearly $a = \log(1 - e^{1/\sigma^*}) - (m^*/\sigma^*)$ and $b = 1/\sigma^*$. See Valdez (2000a).

Example 2 (*Weibull Distribution*)**.** Here, we express the Weibull survival distribution as

$$S_d(x) = \exp\left[-(x/m^*)^{m^*/\sigma^*}\right]. \tag{16}$$

The parameters $m^* > 0$ and $\sigma^* > 0$ are respectively location and dispersion parameters. While this reparameterization has been suggested by Carriere (1992) in the actuarial literature, this distribution is even more widely familiar in survival analysis and reliability theory. Conducting a preliminary investigation of the possible quality of a Weibull model to a survival data is sometimes done visually using the so-called Weibull plot. It can be shown that

$$\begin{aligned}\log(-\log(S_d(x))) &= (-m^*/\sigma^*)\log(m^*) + (-m^*/\sigma^*)\log(x) \\ &= c + d\log(x),\end{aligned} \tag{17}$$

where $c = (-m^*/\sigma^*)\log(m^*)$ and $d = -m^*/\sigma^*$, and is clearly linear in the logarithm of x. Thus, the Weibull plot is the scatter plot of $\log(-\log(S_d(x)))$ against $\log(x)$.

For either the Gompertz or Weibull model, we injected observable covariate characteristics through either (or both) the location parameter m^* or the scale parameter σ^*.

In order to investigate the robustness of the models, other parametric survival models were examined but we found that these did not adequately fit our data. For comprehensive purpose, especially for readers who wish to investigate such other models, we make a list of some in Table 1. As a matter of fact, it has also been suggested in both Carriere (1992) and Valdez (2000a) that mixing some of these survival models provide a better quality fit of survival data over the entire human lifetime.

3. Data characteristics

In our empirical investigation in this article, we analyzed data drawn from a major insurer's portfolio of terminated single life insurance contracts with mortality dates tracked from the US Social Security Administration office. On the aggregate, we drew a randomly selected sub-sample of 65,435 such terminated policies; although the sub-sampling algorithm used results in a random sample, we carefully made the draw in order to preserve the overall characteristics of the insurer's portfolio. To begin, we have three main product classifications herewith labeled PlanTypeP, PlanTypeT and PlanTypeO. PlanTypeP consisted of the traditional participating whole life insurance policies and is approximately 42.4% of the entire sample. PlanTypeP consisted of traditional term insurance products and is approximately 28.0% of the entire sample. For the rest, approximately 29.6%, are PlanTypeO which primarily consisted of conventional Universal Life, although we have very little policies that were term conversion which were grouped into this classification. Term conversion policies are those initially purchased as traditional term contracts that later converted into some form of permanent policies. Similar such proportions for plan types have been observed from the insurer's entire portfolio.

Table 2
Policy characteristics and other observable information.

Categorical variables	Description		Proportions	
PlanType	Type of insurance plan:	PlanTypeP	42.4%	
		PlanTypeT	28.0%	
		PlanTypeO	29.6%	
RiskClass	Insured's assigned risk class:	RiskClass = N	72.0%	
		RiskClass = Y	28.0%	
Sex	Insured's sex:	Male = 1	65.2%	
		Female = 0	34.8%	
Smoker	Smoker class:	Non-smoker = N	66.6%	
		Smoker = S	12.4%	
		Combined = C	21.0%	
Censor	Censoring indicator for death:	Censor = 1	94.6%	
		Censor = 0	5.4%	
Continuous variables		**Minimum**	**Mean**	**Maximum**
IssAge	The policyholder's issue age	0	37.70	89.65
Face Amount	The policy's insured amount	1	213,000	60,000,000
Temp FEAmt	Temporary flat extra amount (per 1000)	0.00	0.08	49.00
Perm FEAmt	Permanent flat extra amount (per 1000)	0.00	0.06	48.00
MEFact	Extra mortality factor	1.00	1.01	4.00
Dates				
IssDate	Policy effective or issue date			
BDate	Insured's date of birth			
WDate	Policy withdrawal or lapse date			
DDate	Insured's date of death, if applicable			

Table 3
Number of policies and average face amount by type of plan, sex and issue age.

Plan Type	Issue age								Total
	Males				Females				
	≤30	30–50	50–70	>70	≤30	30–50	50–70	>70	
PlanTypeP									
Count	6,461	8,476	2,300	100	4,401	4,545	1,374	119	27,776
Face Amount	46,766	152,345	139,624	213,028	35,611	103,401	150,228	213,891	100,605
PlanTypeT									
Count	1,130	9,557	1,963	20	964	4,262	434	3	18,333
Face Amount	323,955	475,092	653,320	1,461,250	168,350	251,603	408,421	425,833	416,264
PlanTypeO									
Count	2,076	7,314	3,091	188	1,516	3,789	1,103	249	19,326
Face Amount	124,896	193,958	203,519	445,704	79,893	133,510	310,929	604,947	181,690

Table 2 provides for a summary of the policy characteristics in our data together with other interesting observable information that we later find useful predictor variables.

To illustrate, gender and smoker categories are included in our data files. It is well known that there are significant mortality differentials between males and females, with females generally living longer than males. We have significantly more males in our data than females, with roughly a ratio of almost 2 to 1. In addition, because of previous medical studies, it has become an acceptable premise that smoking does affect mortality. These findings are again reinforced in the results of our empirical work and our data have roughly five times more non-smokers than smokers. Finally, we also have 21% of our observations classified as combined smoker and non-smoker; these refer to those policies classified as unismokers. Virtually, unismoker policies refer to those insurance contracts with premiums rated regardless of smoking habits.

For each contract observed, we have the policy effective or issue date, the withdrawal date and the date of death, if applicable. Policies with no observable date of death are considered censored observations, with a fixed and known censoring date, herewith being a withheld information to preserve confidentiality. These dates allow us to measure the duration from issue to policy withdrawal, and given this duration of withdrawal, the time of death, if policy is uncensored, or the time from withdrawal till the censoring date, if policy is censored. Our policy records indicate 61,889 of the total 65,435 observations are censored, representing about 95% of the policies in the data.

When insurance policies are underwritten prior to issue, the insurer may find additional or extra hazard, such as certain lifestyle or past illness, for which the insurer may be willing to assume but for obviously with a premium differential or extra cost. Insurers price for these costs with degrees and methods, and according to our records, ours increases the mortality assumption in the premium calculation with an extra mortality factor and/or assesses a flat extra premium on either a temporary or permanent basis. Several of our policies were priced with little or no extra hazard. However, for those that were subjected to such premium differentials, the extra mortality factor used in the premium calculation ranged from as little as just slightly 1% to as high as above 400% of that presumably used for standard policies.

Finally, Table 3 provides an interesting summary of the number of policies together with the average face amounts according to type of plan, gender and issue age. For this purpose, it was meaningful to partition issue age according to 4 groups: less than or equal to 30, between 30 and 50, between 50 and 70, and above 70. Of our total 65,435 policies, we find that the overall average face amount is $212,992. As earlier noted, roughly a bulk of our data are PlanTypeP policies. However, it is interesting to note that PlanTypeT policies tend to have much larger face amounts, with its average more than 4 times the average of that for PlanTypeP policies. The overall average face amount for PlanTypeP is 100,606 while it is 416,264 for PlanTypeT policies. For all types of plan, most issue ages are in the range of 30 through 50 years old, especially so for PlanTypeT. This could either be the result of the choice of the policyholder or that of the insurer. By design, term life insurance products tend to have premiums that exponentially increase with age so that it is not surprising to find fewer policies in the above 50 age categories. What is a little bit surprising is to find fewer policies in the younger age categories; such may be the result of a marketing strategy by our insurer.

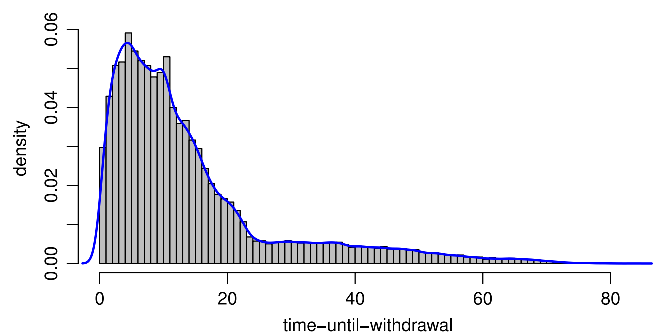


Fig. 2. A frequency histogram of the time-until-withdrawal.

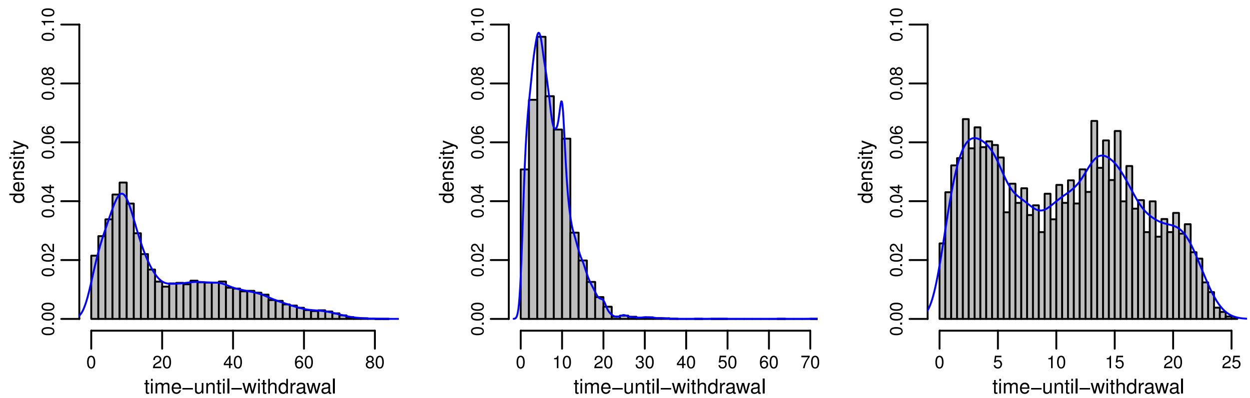
Mortality studies for insurance contracts tend to account for the impact of policy face amount by using them as weights. For our purposes, because our observations are at the policyholder level, we used counts but we do recognize the effect of face amount using them as a covariate characteristic in our parametric models. This is much more flexible as it allows us to directly quantify the effect of any increases in face amount on the survivorship of policyholders.

4. Model calibration results

4.1. Time-until-withdrawal

Prior to fitting the various duration models discussed in Section 2.1, we performed preliminary investigation of the observed distributions of the time-until-withdrawal according to the various available classifications (e.g. Plan Type, Sex, etc.). While it becomes too cumbersome and possibly even overwhelming to show the results of this preliminary investigation for all possible classifications, at best we present this analysis by Plan Type. The time-until-withdrawal has been measured in years from policy issue.

Fig. 2 provides a graphical display of the frequency histogram of the observed time-until-withdrawal for all 65,435 policies. Broadly speaking, we find that policyholders do voluntarily terminate their contracts following the early duration from policy issue. However, upon inspection by policy type as shown in Fig. 3, there is apparently a wide variation. First, a greater proportion of term insurance contracts tends to lapse during the early duration from policy issue; a possible explanation is the exponential increase in premium for such contracts. Second, more permanent contracts



(a) PlanTypeP. (b) PlanTypeT. (c) PlanTypeO.

Fig. 3. A frequency histogram of the time-until-withdrawal by Plan Type.

Table 4
Summary statistics of the time-until-withdrawal.

Plan type	Number	Min	Mean	Median	Max	Std dev
PlanTypeP	27,776	0.08	21.46	14.80	83.75	17.24
PlanTypeT	18,333	0.01	7.34	6.42	70.15	4.83
PlanTypeO	19,326	0.08	10.51	10.62	25.01	6.36
Aggregate	65,435	0.01	14.27	10.01	83.75	13.57

also follow the same pattern but at a much relatively lower rate than term contracts. There is a greater proportion, though, of such contracts to lapse in later years; a possible explanation is the cash value component usually associated with such contracts. Finally, for other types of contracts which primarily consist of Universal Life or similar products, there tends to be more a relatively flat stable proportion of policy lapses across duration; a possible explanation for this is the tendency of these products to be more viewed as savings or investment-type products with relatively less important insurance component.

Table 4 provides basic summary statistics of the time-until-withdrawal according to Plan Type as well as on the aggregate. On the aggregate, the earliest policy termination happened to be about 0.01 of a year, or roughly one week from issue. On the other hand, the latest policy lapse happens after 83.75 years since policy issue. Observe the variation of the summary statistics by type of plan.

In estimating the model parameters, we use maximum likelihood techniques with the log-likelihood function following the form of

$$\log \mathrm{L}(\beta, \theta; t_{w,i}) = \sum_{i=1}^{65,435} \log f_w(t_{w,i}),$$

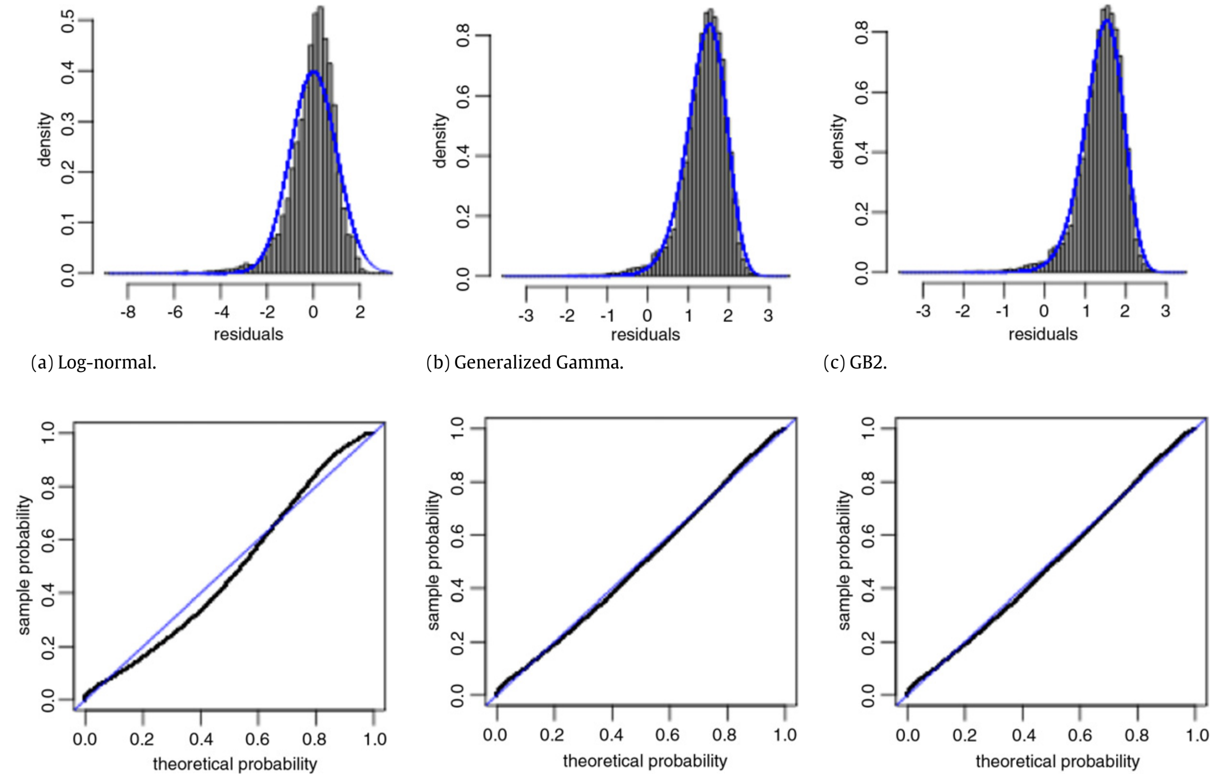
where f_w refers to the density function applicable for the time-until-withdrawal random variable and $t_{w,i}$ refers to the time-until-withdrawal for the ith observation. Here the vector β refers to the set of parameters corresponding to the coefficients in the regression equation for the location parameter μ while θ is the vector of the rest of the parameters applicable to the fitted model.

We fitted three types of parametric models as discussed in Section 2.1: the Log-Normal, the Generalized Gamma and the GB2

Table 5
Maximum likelihood estimates for the various duration models of time-until-withdrawal.

Parameter	Log-Normal	Generalized Gamma	GB2
Regression coefficients			
β_0 (intercept)	2.5534 (0.0263)	1.2138 (0.0419)	3.0034 (0.0238)
β_1 (PlanTypeP)	−0.4022 (0.0071)	−0.1604 (0.0061)	−0.1956 (0.0054)
β_2 (PlanTypeT)	−0.2808 (0.0068)	−0.1422 (0.0060)	−0.2805 (0.0055)
β_5 (RiskClassY)	−0.9787 (0.0063)	−0.6593 (0.0056)	−0.8199 (0.0060)
β_6 (Male)	0.0582 (0.0053)	0.0297 (0.0047)	0.0326 (0.0041)
β_7 (SmokerN)	0.2388 (0.0079)	0.3641 (0.0065)	0.1258 (0.0063)
β_8 (SmokerC)	1.6988 (0.0099)	1.7042 (0.0086)	1.2458 (0.0079)
β_{10} (Face Amount)	−0.0003 (0.0004)*	−0.0027 (0.0003)	−0.0089 (0.0004)
β_{11} (Temp FEAmt)	0.0157 (0.0026)	0.0287 (0.0027)	−0.0258 (0.0020)
β_{12} (Perm FEAmt)	−0.0104 (0.0028)	−0.0167 (0.0023)	−0.0306 (0.0024)
β_{13} (MEFact)	−0.1168 (0.0240)	−0.6373 (0.0162)	−0.1553 (0.0216)
β_{14} (IssAge)	−0.0060 (0.0002)	−0.0092 (0.0002)	−0.0030 (0.0002)
Model specific parameters			
σ	0.6464 (0.0018)	1.2089 (0.0130)	0.2190 (0.0065)
m	–	4.5774 (0.0966)	–
γ_1	–	–	0.4303 (0.0168)
γ_2	–	–	1.2020 (0.0486)
Model fit statistics			
Number of observations	65,435	65,435	65,435
Log-likelihood	−209,054.1	−206,010.2	−201,199.5
Number of parameters	13	14	15
Akaike information criterion	418,134.19	412,048.47	402,428.96

Notes: a. Face amount is re-scaled in 100,000.
b. Standard errors are in parenthesis.
c. An asterisk * identifies 'not significant' at the 5% level.



(a) Log-normal. (b) Generalized Gamma. (c) GB2.

(d) Log-normal. (e) Generalized Gamma. (f) GB2.

Fig. 4. Comparing the quality of fit of the various duration models of time-until-withdrawal.

distribution models. All three models provide enough flexibility so as to capture the observed long tailness as visually demonstrated in Figs. 2 and 3. The estimation of the parameters has been coded using R and is a straightforward procedure. The calibration results for the time-until-withdrawal is numerically summarized in Table 5.

The interpretation of the regression coefficients is rather straightforward; many of these results also do not vary much by the choice of the distribution model. For example, upon inspection of the GB2 model, PlanTypeP and PlanTypeT policies tend to relatively have earlier policy terminations, males tend to lapse later, older issue ages tend to lapse earlier and those policies that are subjected to extra hazard with extra mortality cost tend to lapse earlier.

Fig. 4 provides a graphical display of assessing the quality of the model fit of the various distribution models. For each of the three models considered, we display the histogram together with the parametric fit of the observable errors after taking into account policy characteristics that make the observations heterogeneous. To reinforce the quality of this fit, we provide additionally the corresponding probability–probability (P–P) plots of the observed residuals from each model considered. We find that both Generalized Gamma and GB2, according to these figures, provide reasonably excellent fit; however, the GB2 appears to be a marginally better fit and this is further bolstered by the slightly lower AIC criterion measure displayed in Table 5.

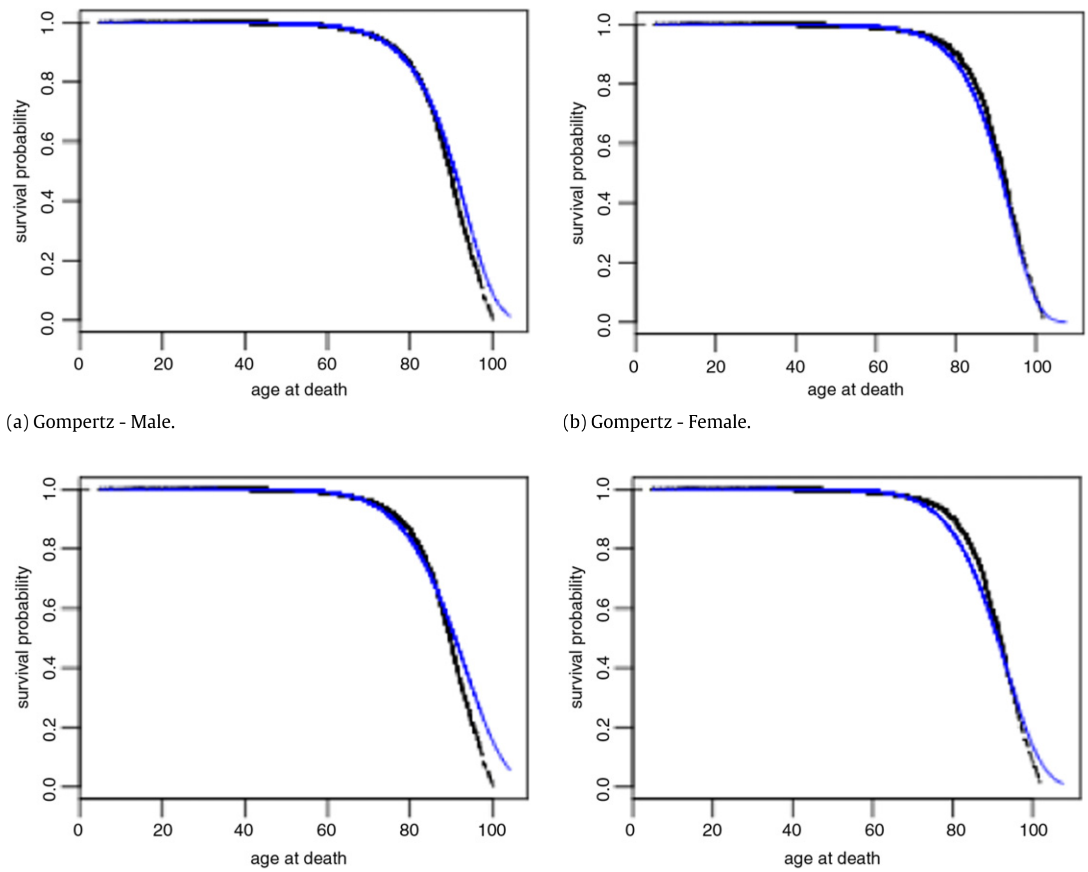
4.2. Age at death

Unlike the time-until-withdrawal, preliminary analysis of the duration from withdrawal till death is much more difficult to perform because of the presence of the censoring of the observations. Table 6 provides the frequency distribution of the mortality status of the policies in the portfolio according to issue age and gender. As shown in this table, there is a strong presence of censoring on the observations. For example, out of the total 42,676 males in the data, we observe only 2480 actual deaths as at the end of the observation period; this represents only about 5.8% of the all males in the data. Similarly, out of the 22,759 females observed, we have 1066 deaths as at the end of the observation period, and this represents less than 4.7% of all females in the data. Furthermore, on the aggregate, we therefore observe only 3546 deaths out of the total 65,435 observations in the data. This is just about 5.4% observed deaths in the data.

Table 6
Number of policies by issue age, sex and mortality status.

Mortality status			
Issue age	Survive	Death	Total
Males			
≤30	8,995	672	9,667
30–50	24,341	1,006	25,347
50–70	6,621	733	7,354
>70	239	69	308
Total	40,196	2,480	42,676
Females			
≤30	6,532	349	6,881
30–50	12,202	394	12,596
50–70	2,653	258	2,911
>70	306	65	371
Total	21,693	1,066	22,759

Maximum likelihood techniques were used to estimate the parameters in the distribution models for the age at death. As discussed in Section 2.2, while we investigated several other parametric models, our analysis resulted in a conclusive decision between the Gompertz and Weibull survival models, both distribution models of which are familiar distributions to actuaries. Our observable data, $(z_i, t_{w,i}, t_{wd,i}, \delta_i)$, consists of the age at issue, the time of withdrawal, the time of death from withdrawal (if applicable), and a



(a) Gompertz - Male.

(b) Gompertz - Female.

(c) Weibull - Male.

(d) Weibull - Female.

Fig. 5. Kaplan–Meier versus fitted survival curves.

censoring variable. The censoring variable δ_i has a value of 1 if censored, that is, the policyholder survived to reach the end of the observation period. Otherwise, it has a value of 0 if the policyholder died during the observation period. Based on this observable data, we constructed the log-likelihood using the result in Eq. (11) as follows:

$$\log \mathrm{L}(m^*, \sigma^*; z_i, t_{w,i}, t_{wd,i}, \delta_i) = \sum_{i=1}^{65,435} \left[(1-\delta_i) \log \frac{f_d(z_i + t_{w,i} + t_{wd,i})}{S_d(z_i + t_{w,i})} + \delta_i \log \frac{S_d(z_i + t_{w,i} + t_{wd,i})}{S_d(z_i + t_{w,i})} \right], \quad (18)$$

where f_d is the corresponding density function for the age at death random variable.

We attempted to fit covariate information to account for policyholder heterogeneity similar to that of the time to withdrawal. However, we found that several of these heterogeneous characteristics did not significantly affect the pattern of mortality once the policy lapsed. In addition, we find that there was no differential between male and female for the location parameter m^*, but such was not the case for the variability parameter σ^*. See Table 7. The estimate for m^* is about 94 years old, both for Gompertz and Weibull models and was not affected by gender. At first glance, we thought that the location estimates appear to be quite high may be a little bit of a surprising result. However, this has been largely a result of the censoring of our observations. As initially indicated, we have only approximately about 5% of our observations that were not censored; the rest were censored. For the uncensored observations, that is where deaths were observed, the median age at death is 81 years old and that the 75th percentile is 88 years old. The maximum age at death observed is slightly above 106 years old. For the censored observations, on the other hand, we found that the median age at the time of censoring is approximately 57 years old and the 75th percentile is 65 years old, with a maximum of 108 years old. All these high ages both for censored and uncensored observations contributed to the high location estimates.

Table 7
Maximum likelihood estimates for the various survivorship models.

Parameter	Gompertz	Weibull
m^*	93.6031(0.1428)	94.2095 (0.1811)
σ^*	6.8420 (0.0975)	8.3039 (0.1337)
$\sigma^* \times$ Male	0.5206 (0.1161)	0.7507 (0.1481)
Model fit statistics		
Number of observations	65,435	65,435
Log-likelihood	−18,264.55	−18,433.82
Number of parameters	3	3
Akaike information criterion	36,535.11	36,873.63

The quality of the fit between the Gompertz and the Weibull models can be visualized in Fig. 5. These figures compare the nonparametric Kaplan–Meier type survival curves against corresponding parametric survivorship curves with parameters calibrated from the data. Kaplan–Meier survival curves do account for the censoring of the observations as in our data. The comparison here is not only between models but also between males and females. Broadly speaking, we find that the Gompertz model, for either the male or female, slightly outperform the Weibull model. This is not at all surprising considering the nature of our observed data; it is well known that the Gompertz model explains a large part of the exponentially increasing pattern of mortality at very high ages. Our observed data is derived from an insured group which consisted, in large sense, of policyholders issued at very high ages and observed for a long time duration. While it is true that we have negligibly few policyholders with very young issue ages, it is quite uncommon to have an insurance coverage at early ages. However, our average issue age in the data has been about 38 years old with a 75th percentile of 46 years old. Indeed, surprisingly, we even have a maximum issue age of approximately 90 years old. Except possibly under special circumstances, insurers typically do not actively seek insurance sale within the very old age market. As can be deduced from Table 6, more than half of the policies have had issue ages around the range of 30–50 years old. Insurance is generally viewed as a financial product that provides economic security against early and premature death particularly for the head of the household; hence, it is not surprising to find the range of issue ages in our data.

5. Implications

In order to understand the consequences of our calibrated models, we examined two material aspects that may be of importance to actuaries. The first one is a deduction of the presence of mortality antiselection. The second one is the financial cost of insurance policy terminations. In this section, we consider these two implications separately.

5.1. Evidence of the presence of mortality selection

In the life insurance industry, mortality antiselection refers to the adverse consequences of the imbalance in the portfolio that may result because policies that do terminate are those with better survival pattern. When there is a presence of antiselection, the insurance company may end up with the spiraling effect of a worsening mortality pattern as a result of policy terminations. Insurance portfolios with worse mortality pattern may have consequences that can negatively impact both the company's balance sheet as well as income statements. Thus, we look for evidence of the presence of such selection.

In analyzing mortality selection, we define what we meant by antiselection. We follow the definition, which are equivalent, considered both in Carriere (1998) and Valdez (2001). We say that there is presence of antiselection at withdrawal in life insurance if

$$S_{d|w}(t_d|t_w) > S_d(t_d), \quad \text{for every } t_d \geq t_w. \tag{19}$$

To interpret definition (19), antiselection is evidently present when the survival pattern of those terminated policies, conditional on all periods of termination, have generally better unconditional survival pattern. To look for evidence in our data, we consider a specific type of a policyholder with the following characteristics: issue age 35, permanent whole life, a non-smoker, male, face amount of 250,000, and not-so-risky with no flat extra charges. We compare the conditional and unconditional survivorship curves for this policyholder if he terminates his contract for different years from issue; here we consider withdrawals for years 2, 4, 6, 8, 10, 15, 20 and 30. The result of this comparison is graphically displayed in Fig. 6.

According to this figure, we clearly find some evidence of mortality antiselection since survivorship of terminated policies are always above those unconditional survivorship curves for all duration of policy termination being consider here. While we certainly can consider an infinitely many more years of policy termination than those being considered, it would be too overwhelming to the reader to see this evident. Suffice to say that we investigated many more years of policy termination, and similar pattern has been observed.

An additional interesting observation that can be made from this figure is the increase in magnitude of the difference between the two curves as policy withdrawals occur in later years. This translates to an even stronger presence of antiselection when policy terminates in later years. This is not at all counterintuitive because policyholders who have had their insurance contracts for a longer period would have to seriously consider their greater risk of mortality as the probability of dying exponentially increases with age; this is even more evident in the parametric form of the Gompertz survival curve.

Finally, it is natural to ask whether the differences between the two survivorship curves we saw in Fig. 6 are statistically significant. To further convinced ourselves, we conducted a formal hypothesis tests based on the Wilcoxon signed-rank sum test to compare the significance of these differences. This is a nonparametric hypothesis tests for examining evidence of significant differences between two related samples, applicably so in this situation. We found that for early withdrawals, in particular for years 2, 4 and 6, the differences are not statistically significant at level $\alpha = 5\%$. This could also be visualized from the graphs. Nevertheless, for policy terminations in later years, specifically for years 8 and beyond, we found the differences to be statistically significant at level $\alpha = 5\%$.

5.2. The financial impact of policy termination

To illustrate how we can quantify the financial consequences of policy terminations, we illustrate this by considering a policyholder with the same characteristics that was considered in the previous section: issue age 35, permanent whole life, a non-smoker, male, face amount of 250,000, and not-so-risky with no flat extra charges. Two types of expenses were assumed in the calculations:

- acquisition expenses: 80 plus 4.5 per 1000 of death benefit; and
- maintenance expenses: 60 plus 3.5 per 1000 of death benefit.

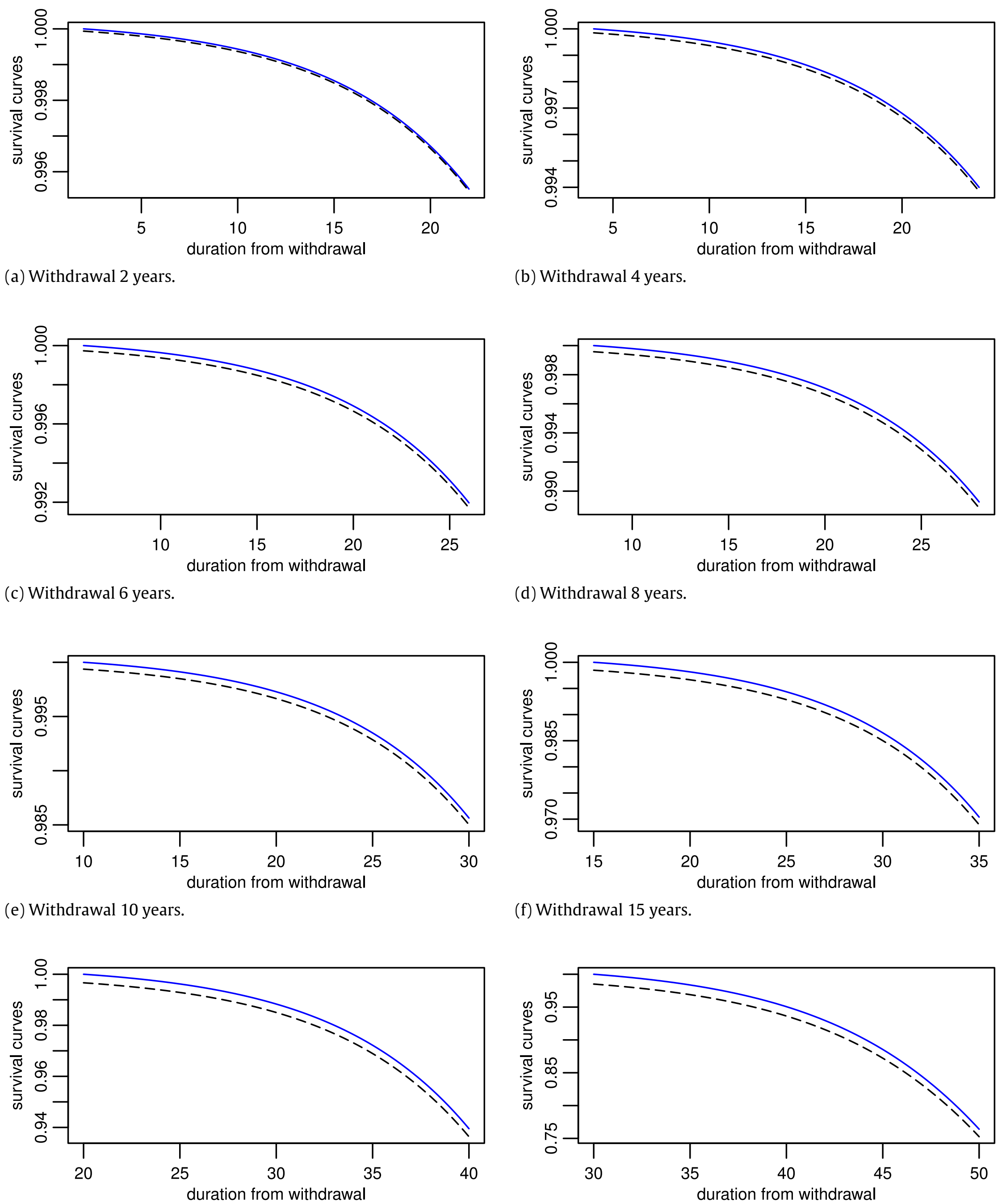
These assumptions have been somewhat drawn from the expense study done in Segal (2002) where he estimated both "the acquisition and maintenance costs associated with life policies". We refer the reader to this article for details if interested. For simplification, we assume that death benefit is paid at the moment of death while premiums, with expenses, occur at a continuous rate throughout each year. Finally, interest rate used for discounting has been set at the constant rate of 5% per year.

In order to investigate the financial impact, first we calculated the premium payable for this policy. We based this calculation on the actuarial equivalence principle, something typically learned in a mathematics for life contingencies course. In this case, the premium has been calculated at the rate of 2010 per annum.

All stochastic components in the calculation process have been done using simulation. The time-until-withdrawal random variables were simulated based on the Generalized Gamma family of distributions. While we said earlier that the GB2 distribution models appear to be marginally better, for simulation purpose, the Generalized Gamma family provides more ease in simulation. The age at death random variables were simulated based on the Gompertz model. To demonstrate for example how to simulate from our Gompertz model as specified in Eq. (12), we can use the inverse transform method. Here, we start with a random number, say U, and generate a Gompertz lifetime, say T, from the following equation:

$$T = \sigma^* \log\Big[1 - (\log(U))e^{m^*/\sigma^*}\Big]. \tag{20}$$

The financial impact is the loss incurred when policy terminates. These loss calculations have been done based on retrospective principles. See Bowers et al. (1986). In effect, we define the



(a) Withdrawal 2 years.

(b) Withdrawal 4 years.

(c) Withdrawal 6 years.

(d) Withdrawal 8 years.

(e) Withdrawal 10 years.

(f) Withdrawal 15 years.

(g) Withdrawal 20 years.

(h) Withdrawal 30 years.

Fig. 6. Comparing survival curves after policy withdrawal for an issue age 35. The smooth curve gives the survival probabilities conditional from policy termination. The dashed curve gives the unconditional survival probabilities. The curves are provided for 20 years from policy termination.

loss at policy termination to be the accumulated values of all past expenses incurred, plus policy reserves, reduced by the accumulated value of all past premiums paid. Once simulations of the random components are done, this process of loss calculation is rather straightforward. We coded the calculations using the R package. This loss is best summarized, first, with a frequency distribution as depicted in Fig. 7. The next paragraph gives summary details of the numerical results.

According to these simulation results, the largest negative loss is −249,500 and the largest positive loss is 248,000. The mean and median losses are, respectively, 1223 and −3128. The 25th percentile is −26,440 while the 75th percentile is 25,610. There is about 54% chance that the loss will be negative and about 46% that the loss is positive. Finally, there is a very slim chance that the loss will be larger than 200,000 but there is a 3% chance it will be above 150,000.

6. Concluding remarks

In this paper, we conducted an empirical investigation of the mortality pattern of terminated policies. We drew a random sample from a follow-up study conducted by a major insurance company which tracked the death pattern of their portfolio of insurance policies. We examined and modeled these life insurance

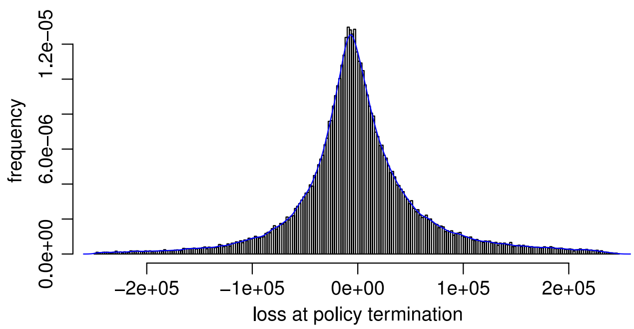


Fig. 7. The frequency histogram of the loss at policy termination.

policy terminations together with their survivorship patterns. We used parametric distribution duration models to calibrate the data observed on time-until-withdrawal and found that the log-location scale class of distributions fit our data very well. This class of distributions is very flexible and is able to accommodate covariate information, through the location parameter, in order to account for the apparent heterogeneity in our data.

The more interesting aspect of our work is studying the survivorship pattern of terminated contracts. The censored nature of mortality data typically presents a challenge when calibrating the age at death data. While we investigated several classes of parametric mortality distribution models, we narrowed our choice between the Gompertz and the Weibull survival models. We found that while both provide quality fit to the data, the Gompertz model appeared to marginally outperform the Weibull model. Just as with the class of duration models we investigated for the time-until-withdrawal, we injected covariate information directly through the location and scale parameters; however, we found very little statistical evidence of heterogeneity in the mortality pattern. Indeed, even surprisingly, gender did not statistically affect the average age at death, but it did so on the variation of this age at death.

Furthermore, we examined the actuarial implications of our model calibration. We found that the data provides support for evidence of mortality antiselection. This indicates that policies that do terminate generally tend to have better survivorship pattern than those who do not. However, we also discovered that the difference in the survivorship pattern is affected by when the policy terminates. Generally speaking, there is little or no statistical significance for early policy terminations, in particular, for terminations before 6 years from policy issue. Beyond 6 years, we found stronger evidence of antiselection and this antiselection increases with later terminations.

Finally, we assessed the financial effect of policy terminations by examining the loss that would have been incurred when the policy terminates. The loss usually consists of unrecoverable expenses incurred at policy issue (it is well known that at policy issue, acquisition expenses are too large relative to the premium collected), net premium reserve that is usually released when the policy terminates, and the loss of future uncollected premiums. To perform the investigation, we examined a particular policy with specified characteristics as a case study. We found that there is about a 50–50 chance of a negative and positive loss when policy terminates.

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