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Pricing for the Volatility Risk of Traditional Actuarial Risks

by Jay Vadiveloo and Charles Vinsonhaler

Editor's Note: The section's Corporate and Chief Actuaries listserve would be an appropriate forum for discussing concepts in this article.

1. Introduction

Actuarial literature has placed a lot of emphasis on the interest rate risk and asset liability management. For the most part, the traditional actuarial risks such as mortality, morbidity and lapses, have been relegated to experience studies and experience tracking reports. The volatility of interest rates and its impact on asset liability management has been analyzed by looking at complex, stochastically-generated interest rate scenarios and their corresponding impact on a company's future earnings. Risk-based capital formulas and asset-adequacy analysis all seek to quantify and understand this risk, and this kind of analysis has involved the finest of actuarial minds and a large part of the actuarial consulting practice.

On the other hand, the analysis of the traditional actuarial risks, which is the foundation of actuarial science, has pretty much stayed in the deterministic plane. Experience studies and the construction of experience tables are studied only at the early stages of the actuarial exams, and are certainly not one of the sought-after areas for practicing actuaries.

This paper will do the following:

- Explain the reasons for the lack of evolution in the analysis of these traditional actuarial risks.
- Explain why, for certain product designs and markets, understanding, measuring and managing the volatility of these traditional actuarial risks are critical to the financial success of such businesses.
- Provide a general definition of the volatility risk for these traditional actuarial risks.
- Provide a general stochastic methodology to measure this volatility risk and incorporate it in pricing and reserving.
- Provide a general technique to develop a practical, deterministic, formula-based equivalence to this stochastic methodology.
- Provide examples of these formula-based approximations to measure the volatility risk for three insurance products.

2. Scope of paper

The traditional actuarial risks whose volatilitus are being analyzed in this paper are mortality, morbidity and lapse risks. Even though this paper is titled as "pricing for the volatility risk," it is easily extended to reserving or setting capital standards for this risk. In fact, depending on the particular product being analyzed, it may be more appropriate to indirectly price for the volatility risk by first determining the risk-adjusted benefit reserve, risk-adjusting the appropriate actuarial rates and then determining the risk adjusted benefit premium. One of the examples in the final section of this article demonstrates this.

The authors want to emphasize that this paper is analyzing just the volatility risk, and not the misstatement risk, where the underlying base risk has been wrongly estimated. Experience studies, good underwriting practices, claims management and experience tracking are the best ways to avoid a complete mis-statement of the risk. However, the volatility risk doesn't go away and can be significant, even if the base risks are properly stated.

3. Reasons for lack of development in the analysis of traditional actuarial risks.

As mentioned in the introduction, traditional actuarial risks are analyzed in experience studies and tracked in experience reports. These individual or inter-company studies form the basis of pricing and projection models involving these risks. In order to recognize fluctuations from historical experience, some provisions may be made for adverse deviation. These provisions, which are the only attempt to address the volatility risk in these traditional actuarial risks, are usually arbitrary in nature and have no statistical basis.

In many product designs and markets, this approach to pricing these traditional actuarial risks is adequate. These are product designs and markets where the pooling principle applies and the Central Limit Theorem assures us that the standard deviation (i.e. volatility parameter) of the sample mean (i.e. average benefit premium) converges to zero. In these situations, properly constructed experience studies to get a good estimate of the risk factors (i.e. mortality, morbidity and lapse rates) is the correct approach. Building in some conservatism to these estimates is a prudent way to cover the mis-statement risk, and the volatility risk is non-existent or immaterial.

4. Significance of the volatility risk of traditional actuarial risks.

The pooling principle breaks down under one or more of the following conditions:

A. The block of business affected by these risks is not large enough so that the volatility of the average premium does not converge to zero.[1]

B. The business block is large enough, but the benefit obligations are sufficiently large and varied to offset the convergence to zero caused by the volume effect.

C. The risk factors themselves are imprecise (e.g. old-age mortality and morbidity, or substandard risks) and this generates enough volatility to overcome the convergence to zero by the volume effect.[2]

There are several product designs and markets where one or more of the above conditions could hold. The second-to-die product is an example where all three conditions could hold. The disability income market typically satisfies the first two conditions, reinsurance pricing for substandard mortality and long-term care pricing would involve condition C, and so on. In all these cases, the volatility risk of these traditional actuarial risks can have a significant impact on the earnings of a company, and it is critical that this is reflected in the pricing, reserving and required surplus models for these products.

5. Definition of the volatility risk of traditional actuarial risks.

Let R = { r_1, r_2,....r_n} be a set of risk factors for a given risk.

e.g. R = set of select and ultimate mortality rates for an individual age (x)

or R = set of incidence and termination rates of disability.

Let P(R) be an appropriate present value random variable.

e.g. P(R) = loss-at-issue random variable (i.e. pricing random variable)
= present value of benefits less present value of premiums, at issue.

or P(R) = prospective loss random variable (i.e. reserving random variable)
= present value of future benefits less present value of future premiums, given (x) survives to (x + t).

or P(R) = present value of distributable earnings
(i.e. embedded value random variable).

Current practice is to use the expected value of P(R), E[P(R)], as the estimate of this present value random variable, or E[P(R*)] where R* is R with some provision for adverse deviation.

Consider the distribution of P based on all possible realizations of R. Rank these values and denote them as P1, P2, …PN. Then, for a given confidence level of (100*c)%, PcN or P[(1-c)N] is the appropriate risk adjusted present value random variable. For example, for the pricing random variable, P[(1-c)N] would be the risk adjusted estimate, whereas for the embedded value random variable, PcN, would be the appropriate risk adjusted estimate. The absolute difference between the risk adjusted estimate and the expected value of P is the volatility risk factor at a given confidence level c.

continued on page 14

In many product designs and markets, this approach to pricing these traditional actuarial risks is adequate.

There are several practical limitations to this process of determining the risk adjusted present value random variable at a given confidence level of c.

- In many models, the present value random variable will be impacted by more than one risk set. i.e. P = P(R, S, T,….) for different risk sets R, S, T,……
  e.g. the pricing random variable for long-term care will be impacted by the various combinations of lapse, morbidity and mortality realizations. Then the set of all possible values of P may be impossible to enumerate.

- Even if there was only one risk factor, the risk adjusted present value random variable should be determined for a group of contracts that is being priced or reserved for.
  e.g. P is the average present value random variable for the group of contracts. So if there are N possible realizations of P for a single contract, there will be N possible realizations of the average present value random variable for n contracts. Even for small values of n, this is just not practical to evaluate.

The stochastic simulation is the theoretically best way to determine the risk-adjusted present value random variable.

6. STOCHASTIC METHODOLOGY TO MEASURE THE VOLATILITY RISK OF TRADITIONAL ACTUARIAL RISKS.

Let P(R,S,T,….) be an appropriate average present value random variable for n contracts, which is impacted by the sets of risk factors R, S, T,…….

Generate N realizations of P by stochastic simulations of R, S, T, ….

Rank the possible values of P, denoted by P1, P2, …PN, and for a given confidence level of c, the risk adjusted estimate of the average present value random variable is PcN or P[(1-c)N].

The following should be noted about this methodology:

- The model can be made as complex and flexible as the actuary desires, and only requires a good random number generator, strong programming skills and a high-speed computer.

- The more complex the model and the more varied the number of risk factors, the greater the number of simulations required to approximate the true underlying distribution of the present value random variable P.

- If there are several risk factors in the model, an assumption should be made about the order of occurrence of these risks in generating the random numbers. For example, if the three risk factors—lapse, morbidity and death—are assumed to occur in that order, the lapse rate is first randomly generated, followed by the incidence rate of disability if the contract did not lapse, and followed by the mortality rate if the incidence rate of disability did not occur.

- The model can incorporate dynamic relationships between the risk factors. For example, as lapses occur in a block of lives being modeled, the mortality rate of the persisting block can be systematically increased if an assumption is made that the healthy lives have a greater propensity of lapsing. For a second-to-die model, the mortality rate of the survivor can be increased upon the first death to replicate the contagion effect.

- To simulate condition C in section 4, the imprecision of the risk factors can be captured by using an interval estimate for the risk factor. For example, if an old age mortality rate q is imprecise and could vary from q to (1 + s)q, s>0, then a uniform random number could first be selected between q and (1+s)q to determine the underlying mortality rate, and then this under lying mortality rate is used in the simulation. Of course, modeling this imprecision in the risk factors increases both the expected value and volatility risk of the present value random variable, as should be the case.

7. DETERMINISTIC APPROXIMATIONS TO THE RISK ADJUSTED PRESENT VALUE RANDOM VARIABLE.

The stochastic simulation is theoretically the best way to determine the risk-adjusted present value random variable. However, even with state– of–the–art technology, the processing time becomes unmanageable as the block of business starts to grow. To illustrate, a block of 1000 lives would require one million simulations to generate 1000 realizations of the average present value random variable. The need for a deterministic approximation is clearly necessary.

The general formula to estimate the risk adjusted average present value random variable P for a block of n identical contracts is :

$$E(P) + z(1-c)\,STD(P)$$

or, $E(P) - z(1\text{-}c)\ STD(P)$

where $E(P)$ = the traditional approach to estimating the average present value random variable, without any provision for adverse deviations, $z_{(1\text{-}c)}$ = (1-c) percentile value of the standard normal random variable.

$STD(P)$ = standard deviation of the average present value random variable.

Since the n contracts are identical,

$$E(P) = \mu$$

where μ = expected present value for a single contract.

$$STD(P) = \sigma/\ n$$

where

σ = standard deviation of the present value for a *single* contract.

The formula can easily be modified when the contracts are distinct. Now,

$$E(P) = \frac{[\mu_1+\mu_2+ \dots \mu_n]}{n}$$

where

μ_i = expected present value for contract i

$Var(P)$ = variance of the average present value random variable

$$= \frac{[\sigma_1^2+\sigma_2^2+ \dots +\sigma_n^2]}{n^2}$$

where

σ_i^2= variance of the present value for contract i

and

$$STD(P)= [Var(P)]^{0.5}$$

More typically, a company would break up its block of business into k groups, where the contracts within a particular group are deemed identical. Then the calculation of the risk adjusted average present value random variable would involve stochastically modeling only k distinct contracts and appropriately modifying the general formula.

The following should be noted about this general formula to calculate the risk adjusted average present value random variable.

- The only stochastic simulation needed is for individual contracts, versus modeling a group of contracts. This allows for spreadsheet models to be used to determine the risk adjusted present value random variable.

- Only the standard deviation of the present value random variable for an individual contract needs to be estimated. The expected value is what is currently calculated in pricing or reserving using best guess estimates.

- In some models, the standard deviation for a single policy can be determined analytically, e.g. pricing for a standard insurance or annuity contract. Then the calculation of the volatility risk in pricing or reserving can be programmed and determined on a seriatim basis.

- In cases where the standard deviation cannot be determined analytically, it has to be estimated. Stochastic simulation of the present

continued on page 16

value random variable for a single policy can be used to estimate the standard deviation parameter. To estimate the volatility risk for a block of such policies, the best technique is to form policy groupings, estimate the standard deviation of the group by simulation, and then determine the overall volatility using the general formula.

- Another technique is to simulate a function of the standard deviation of the present value random variable, and examine its behavior for different characteristics of a single policy. Using statistical techniques, an analytical approximation to the standard deviation function can be developed for any policy. Then the volatility risk for a block of policies can be calculated on a seriation basis.

8. EXAMPLES

The three examples that follow are based on the research work of three graduate students in the University of Connecticut Ph.D. Actuarial Science Program, under the supervision of the authors. All three dissertations have been submitted for publication in various actuarial journals, hence the authors will not go into too much details on the analysis and results.

The first example looks at the volatility risk for a joint and survivor immediate annuity. The analytical solution for the standard deviation of the present value random variable for a single life immediate annuity is well described in the Actuarial Mathematics text, but for a joint and survivor product, with benefits changing upon the first death, the solution is not that clear. Included in the research work is an analytical formula for determining the standard deviation of a joint and survivor immediate annuity, whose benefits could change depending on whether both lives are living or on the particular life that survives.

To illustrate the volatility impact, the net single benefit premium must be increased by five percent for a group of 100 joint and survivor lives of 60-year-old males and females, where the benefit payment amount does not change upon the first death.

The second example considers the volatility risk for an individual disability income policy. The simulation modeling work done in the research shows that the best approach is to start with developing the risk-adjusted claim reserve. Using statistical and actuarial techniques, the research develops a deterministic, analytical approach to calculate the risk-adjusted claim reserve for an arbitrary block of DI claims at a given confidence level. The research then shows how to develop risk adjusted claim costs, risk-adjusted active life reserves and risk-adjusted premiums.

To illustrate the volatility risk for a group of 360 newly-disabled lives at age 45, the claim reserve has to be increased by 18 percent for a 30-day waiting period, benefits to age 65, to achieve a 90 percent confidence level. For the same group with a 90-day waiting period, the volatility risk factor is only 11 percent, demonstrating that the claim reserve volatility decreases as the waiting period increases.

The final example looks at the volatility risk for two typical long-term care product designs in today's marketplace. One is the stand-alone long-term care product that pays out long-term care benefits when the policyholder qualifies to receive such benefits. The other product design has the long-term care benefit as a rider to a life insurance contract. Here the long-term care benefits can be viewed as early payments of the death benefit, and the payment upon death is the difference between the face amount of the policy and the total long-term care benefits paid to date. The cost of the rider long-term care design is effectively the time value of money arising from two different streams of benefit payments, with or without the rider.

The simulation modeling work done in the research shows that the ratio of the standard deviation to the expected value of the present value random variable has a nice functional form. Using stepwise regression techniques, an analytical formula for this ratio is developed as a function of the age of the insured, benefit level of the long-term care coverage and incidence rate of disability.

The results are quite interesting. While the rider cost is significantly smaller than the stand-alone benefit, the volatility risk for the rider long-term care design is higher (as a percent of the base premium) than the stand-alone design. For example, for a block of 5,000 males, issue age 64, at a 90 percent confidence level, the stand-alone long-term care product requires a 4.7 percent increase in the average net single benefit premium and a 22.9 percent increase in the average annual benefit premium to cover the volatility risk. In contrast, the rider long-term care product design requires a 6.6 percent increase in the average net single benefit premium and 30.1 percent increase in the average annual benefit premium.

The following should be noted about the three examples described above:

- All three use different techniques to arrive at the deterministic, analytical approach to measure the volatility risk. They all involve sophisticated modeling and creative mathematical analysis, which is what our actuarial training and experience equips us to do.

- All three approaches were tested against a full blown stochastic simulation, and the results are very close.

- All three approaches end up with an algorithm that can be implemented by any company in these lines of business.

9. Conclusions

It is the hope of the authors that this article will stir up the actuarial community to pursue this kind of analysis for other product designs and markets. This is research in the traditional areas of expertise of actuarial science, but now carried to a higher level, utilizing sophisticated stochastic modeling and statistical techniques of analysis. While this paper offers guidelines and a structure about how to price and reserve for the volatility risk of traditional actuarial risks, it is not a cookbook formula that can be applied to any product design or market. The three examples described in this paper show how unique the deterministic approximations are, and hence there is really no limit to the future research that can be done in this area.

The authors wish to emphasize the following:

- Experience studies, experience tracking and good claims management processes remain a critical function and is part of the total analysis of these actuarial risks, including the volatility component. Since the volatility risk analysis utilizes best-guess estimates of these risks, good experience studies and tracking are necessary to ensure that these risks are not misstated.

- Besides developing risk-adjusted pricing and reserving formulas, the ideas in the paper can also be utilized to determine the dividends that should be retained for the volatility risk in a mutual company, the basis for changing premiums for guaranteed renewable contracts, and for solvency or capital standards analysis.

As interest continues to grow in this area, the authors are confident that creative actuarial minds will find other uses and implications for this kind of analysis.

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Jay Vadiveloo, FSA, MAAA, is senior manager at Deloitte & Touche LLP in Hartford, CT. He can be reached at jvadiveloo@deloitte.com.

Charles Vinsonhaler, ASA, is a professor at the University of Connecticut in Storrs, CT. He can be reached at vinson@math.uconn.edu.