Voya Equity Indexed Annuity Reserving APL Conversion

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Contents

1. Algorithm

This document summarized the applied actuarial method of calculating three types of EIA reserves (GAAP, STAT, and Tax).

1.1 General Applicable Principle

Since this method is applied to EIA (Equity Indexed Annuity) reserving, each policy has multiple records of premium deposits (or transitions) with different transaction dates. The total reserve of a policy is the sum of reserves based on each transaction (or deposit) of the policy. The order of records matters because it is assumed that withdrawals follow the rule of FIFO (first-in, first-out). Generally, the initial projection period is assumed to be 20 years. However, for different reserves, projection period may be applied in different ways based on some specific assumptions. For example, statutory projection period is determined by issue date, whereas GAAP projection period is based on transaction date. The general projection period, 20, is a cap for both of the two reserve. The following table shows the summary of the projection period in different reserve calculations.

Summary of Projection Period

Reserves	Variables	Calculation Method of Projection Period
GAAP	Host as of issue	For each transaction, project the future 20 years cash flows (actually, the projection time period is 15 years since there is an assumption of 100% lapse rate after 15 years). Discount each future cash flow back to the corresponding transaction date.
	Host as of valuation date	Accumulate the host as of issue to the valuation date at an assumed interest rate.
	Embed option	Project cash flows from the valuation date for the future (1 + 20 - (valuation date-transaction date)) years after valuation date. Calculate THE present value of each future cash flow at the valuation date.
STAT/TAX	CARVM	Project account value from the valuation date for the future (22 - (valuation date-transaction date)) years for each transaction. Calculate present value of each future account value at the valuation date.
	CSV	Accumulate each net deposit from the transaction date to the valuation date.

For the account value at the valuation date, firstly, each actual deposit is accumulated by an accumulation factor, and then less withdraw, thirdly, adjusted by assumed interest rate 0.06, from the transaction date to the sweep date. Detailed calculation can be found in 3.11, where REVAMT represents the account value at the valuation date. The accumulation factor can be looked up from the table SWPRATES, which is supposed to be updated quarterly. The guaranteed account value at the valuation date is the 90% of each deposit accumulated from the corresponding transaction date to the valuation date at an assumed interest rate, 0.03. Detailed calculation can be found in 3.13, where GMV represents the guaranteed account value.

The market value of the option for different reserve uses different interest rates. Option value in STAT reserve is accumulated by STATRATES. The option value in TAX reserve is calculated by TAXRATES. The option value in GAAP reserve is based on forward rate.

1.2 US GAAP Reserve

The total US GAAP Reserve is the sum of the embedded option reserve as of valuation date (FAS157) and the host instrument value as of the valuation date. The host as of valuation date equals to the host as of issue date accumulated by an implied interest rate during the projection period.

1) Initial Bifurcation of Each Premium Deposit

The account value is net deposit accumulated by the adjusted forward rate, which is BUD*(1+forward rate). BUD is an assumed rate 0.04. The forward rate is determined by the risk-free rate as of the greater of transaction date and 9/1/2009.The risk-free rate can be searched from the table SPOTRATES.

The embedded option is the account value less the greater of net deposit and non-forfeiture value. Liability cash flows are interest generated by the embedded option at the decrement rate during the projection duration. The projection duration is 20 years from the transaction date. The current decrement rates assume that the policy lapses in the 15th policy year, which is determined by TRT, a hard-coded variable in the original APL program.

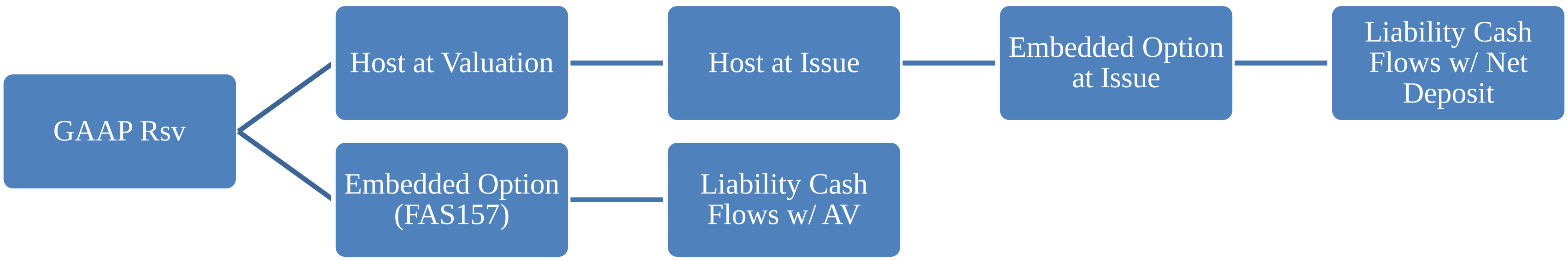
The embedded derivative reserve is the sum of discounted liability cash flows at the risk-free rate as of the transaction date. Please see more detailed calculations in the formula 3.20. The host as of issue is the net deposit less the embedded option reserve. The detailed calculation method is mentioned in the formula 3.21. The implied host accretion rate is the interest rate that is used to accumulate the host as of issue to non-forfeiture value. For more details, please see the formula 3.23.

2) Embedded Derivative and Host Value as of Valuation Date

Account value is the general account value accumulated by adjusted forward rate, which is BRT*(1+forward rate). BRT is $\frac{\text{Market value of option}}{\text{Orig.Account value}}$ in duration 1 and 0.04 afterwards. Forward rate is determined by risk-free rate searched from table SPOTRATES as of valuation date. The market value of option is determined by the difference between option P and option S. Option P and S are calculated by Black-Scholes formula. The difference between the two option is the strike price. The price of option P is higher, since it has lower strike price, S&P 500. Whereas, the strike price of option S is $\left(1+\frac{\text{CAP Rate}}{\text{PARTICIPATE Rate}}\right)\times$ S&P 500. Please see more details in formula 3.16.

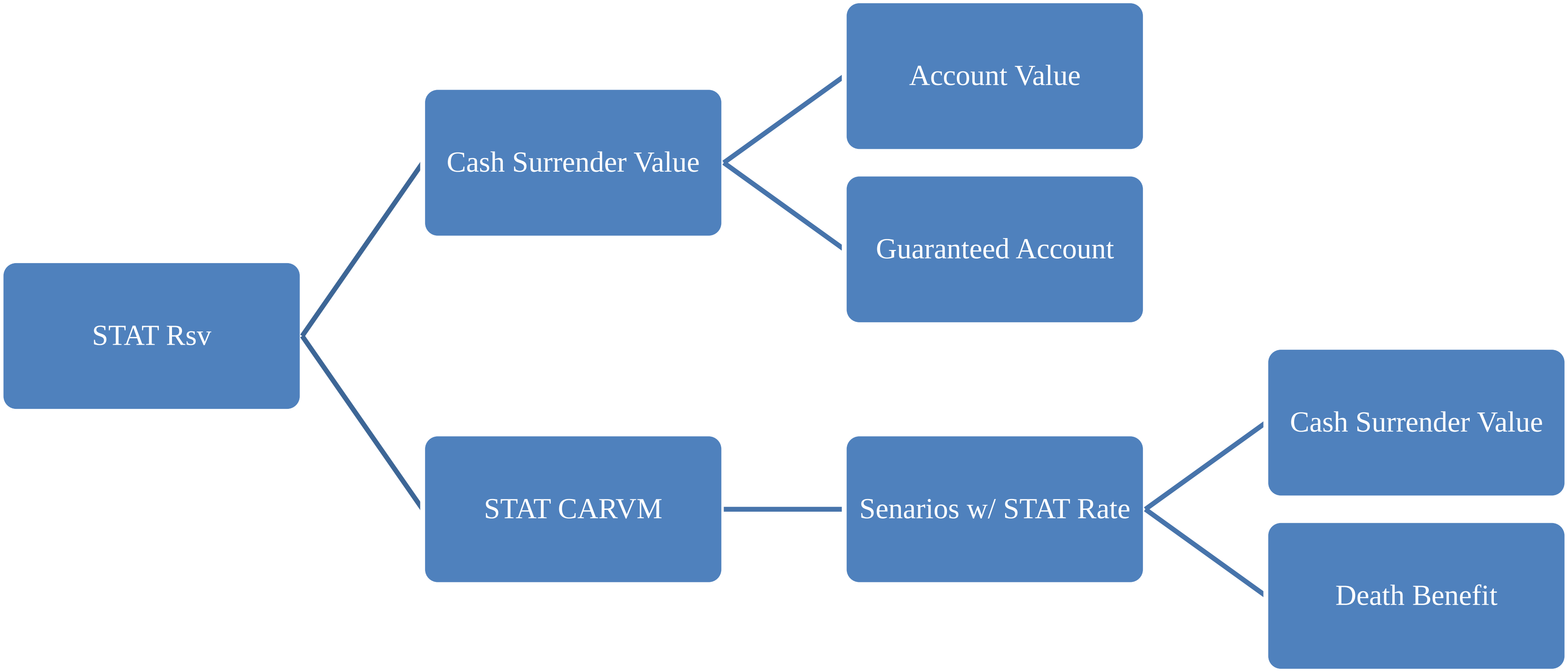
The embedded option is the account value less the greater of net deposit and non-forfeiture value. The liability cash flows are interest generated the embedded option in the projection duration. Projection duration is a remaining term, which can be explained in the following example. A policy was issued on 08/25/1998. Policyholder made a deposit on 9/25/1998. Valuation date is 3/31/2018. Then the valuation duration is 20, ceiling ((Val Date-Issue Date)/365). Transaction duration is 20, ceiling ((Val Date-Trx Date)/365. Projection duration is 2, 1+max (20-Val Dur+Trx Dur, 1)-floor ((Val Date-Trx Date)/365). The current decrement rates assume that the policy lapses in the 15th policy year, which is determined by TRT, a hard-coded variable in APL.

Embedded derivative reserve is the sum of discounted liability cash flows at risk free rate as of valuation date. Please see more detailed calculation in formula 3.24. Host as of valuation is the accumulated initial host during the period between transaction and valuation date at rate of implied interest rate. The detailed calculation is mentioned in formula 3.25.

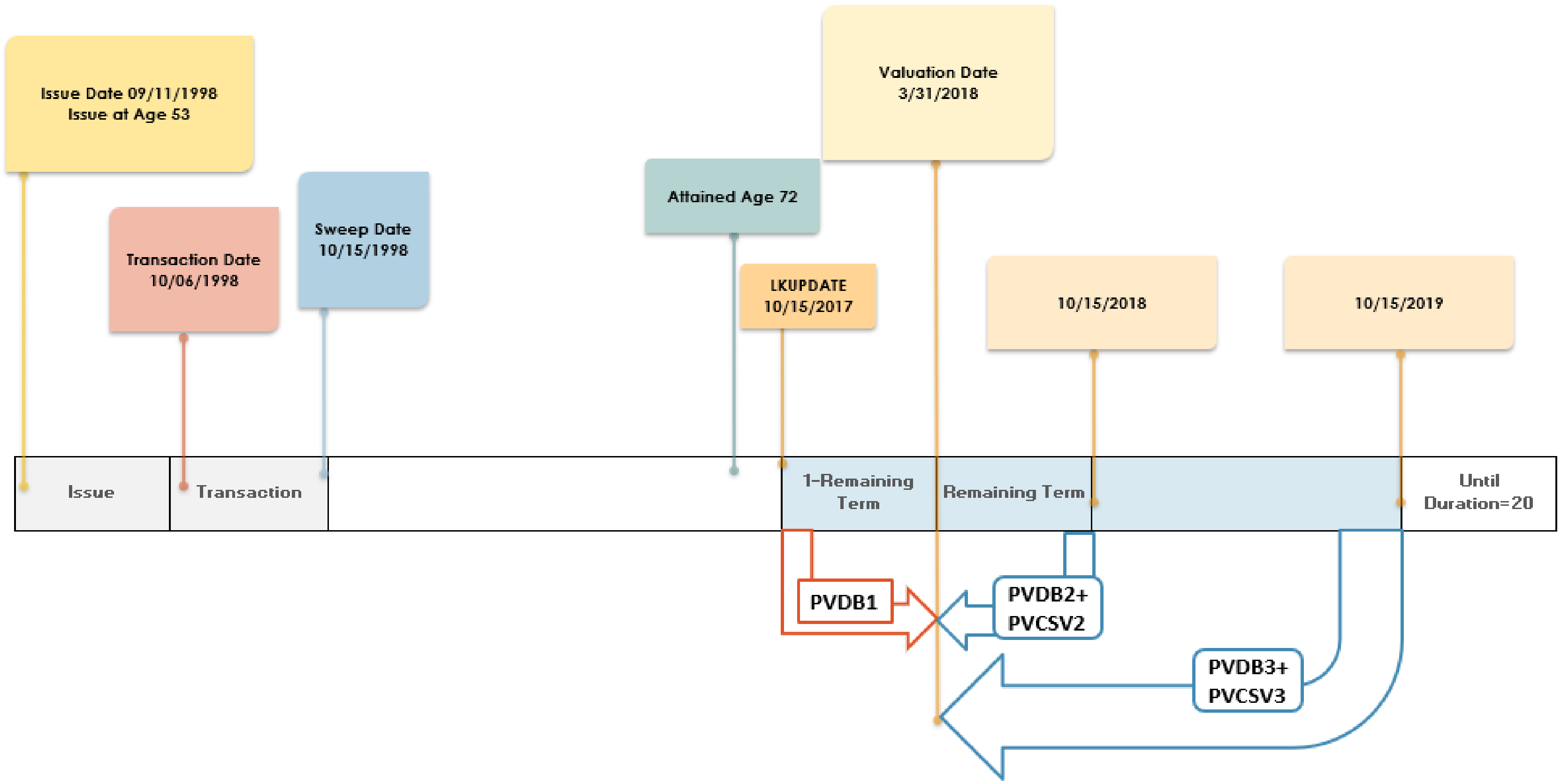

GAAP Rsv
Host at Valuation
Host at Issue
Embedded Option at Issue
Liability Cash Flows w/ Net Deposit
Embedded Option (FAS157)
Liability Cash Flows w/ AV

1.3 Statutory Reserve

Statutory reserve is the maximum of cash surrender value and statutory reserve using CARVM. CARVM is based on scenario tests, calculating the present value of the future death benefit and present value of the future cash surrender value under different assumed durations and then choose the maximum value from the different scenarios. Cash surrender value is calculated as the projected account value less surrender cost, where surrender cost is an assumption searched from table SC. Projected account value is the sum of account value and STAT market value of option accumulated at 0.03 during projection period. Projection period is 22 less the policy year. Please refer formula 3.15 for detailed calculation information.


STAT Rsv
Cash Surrender Value
Account Value
Guaranteed Account
STAT CARVM
Senarios w/ STAT Rate
Cash Surrender Value
Death Benefit

Statutory reserve in CARVM is explained in the following example.



$$\text{PVDB} = \max(\text{Guaranteed}, \text{Account Value} + \text{Option}) \times (q_x \times (1+i)^{1-t} + {}_{1|}q_x v^t + {}_{2|}q_{73} v^{t+1} \ldots)$$

$$\text{PVCSV} = \max(\text{Guaranteed}, \text{Account Value} + \text{Option}) \times {}_n p_x \times (1+i)^{\text{Duration}-20-\text{RMNGTERM}}$$

In our example, the issue date is 9/11/1998 and valuation date is 3/31/2018. Then the duration is 20.

$$\text{STAT\_CARVM} = \max(\text{PVDB1}, \text{PVDB2} + \text{PVCSV2})$$

Scenario 1

$$\text{PVDB1} = \max[\text{REVPREM} \times 0.9 \times 1.03^{19}, \text{REVAMT}] \times q_{73} \times (1 + \text{STATRT})^{1-\text{RMNGTERM}}$$

Scenario 2

$$\text{PVDB2} = \text{PVDB1} + \max[\text{REVPREM} \times 0.9 \times 1.03^{20}, \text{REVAMT} + \text{NETMV\_S}] \times {}_{1|}q_{73} \times (1 + \text{STATRT})^{-\text{RMNGTERM}}$$

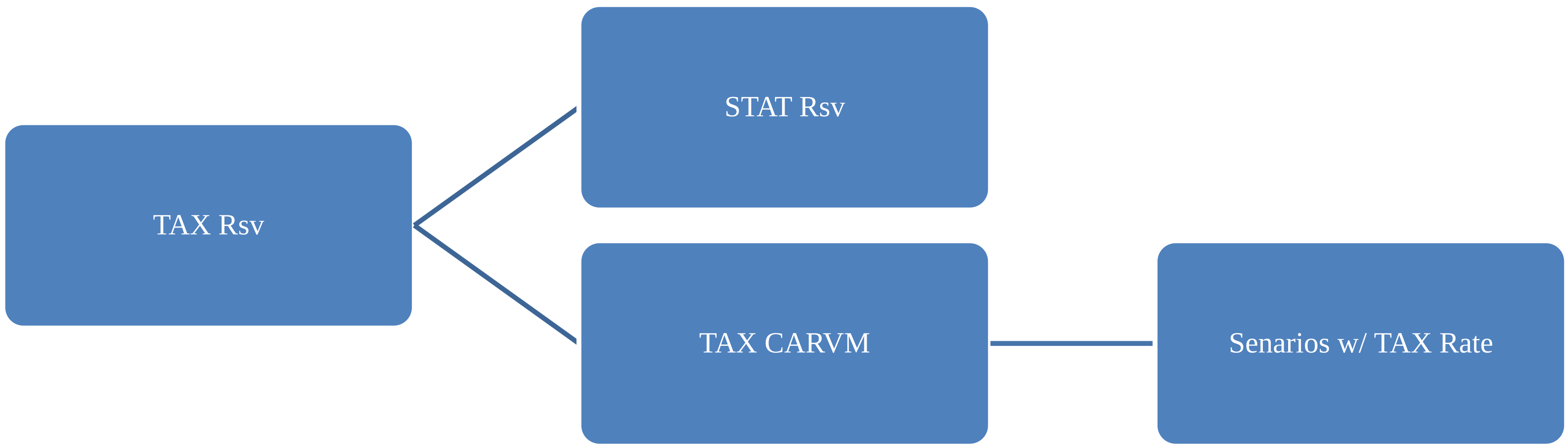
$$\text{PVCSV2} = \max[\text{REVPREM} \times 0.9 \times 1.03^{19}, \text{REVAMT} + \text{NETMV\_S}] \times {}_2p_{73} \times (1 + \text{STATRT})^{-\text{RMNGTERM}}$$

$$\text{NETMV\_S} = \text{NETMV} \times (1 + \text{STATRT})^{\text{RMNGTERM}}$$

$$\text{RMNGTERM} = \max(1 - \frac{\text{VALDATE} - \text{LKUPDT} + 1}{365}, 0.0001)$$

1.4 Tax Reserve

Tax reserve is the minimum of statutory reserve and tax reserve using CARVM. The only difference between tax CARVM and stat CARVM is the interest rate, where tax rate is used in tax CARVM.



2. Input

In the input text file, there are 17 columns, but only 15 of them will be used into the reserving calculation.

1) POLICY: Policy number (one column)
2) ISSDT/ISSDATE: issue date (three columns: year, month, date)
3) ISSAGE: issue age (one column)
4) SEX: gender (one column)
5) LOB: line of business (one column)
6) TRXDT/TRXDATE: transaction date (three columns: year, month, date)
7) SWPDT/SWPDATE: sweep date (one column)
8) SWPDAY: sweep date (one column)
9) INITPREM: initial premium (one column)
10) TOTWD: total withdraw (one column). For each policy there is only one total withdraw, which is just the sum up of all the withdraws that already happened. This number is repeated in each record of the policy.
11) TOTAV: total account value (one column). This number is as of sweep date directly. In the reserving calculation, we still need to calculate as of transaction date.
12) Two state abbreviation columns, issue state and resident state.

3. Calculation and Output

In the output text file, there are 28 columns. 8 of them come from the input file:

1) Policy = Input 1) POLICY
2) LOB = Input 5) LOB
3) Issue Age = Input 3) ISSAGE
4) Sex = Input 4) SEX
5) Issue Date = Input 2) ISSDT/ISSDATE
6) Trans Date = Input 6) TRXDT/TRXDATE
7) Initial Deposit = Input 9) INITPREM

28) Vantage AV = Input 11) TOTAV

So, there are still 20 columns we need to calculate (highlighted ones were already completed):

8) WD from Dep/WDAMT

$$\text{WDAMT} = \max(\text{INITPREM} - \text{REVPREM}, 0)$$

$$\text{REVPREM}(t^{1}) = \begin{cases} \text{INITPREM}(t) - \text{TOTWD}(t), t = 1 \\ \text{INITPREM}(t) - \text{RMNGWD0}(t-1), t > 1 \end{cases}$$

[1] t: Index of each record (or row) for each policy, t =1 means the first and earliest transaction for each policy

$$\mathrm{RMNGWD0(t)} = \begin{cases} \mathrm{TOTWD(t) - INITPREM(t), t = 1} \\ \mathrm{RMNGWD0(t-1) - INITPREM(t), t > 1} \end{cases}$$

9) Dep Net of WD/REVPREM

Please see bullet point 8

10) WD from AV/AV WD

$$\mathrm{AV\ WD = max\ (CURAMT - REVAMT, 0)}$$

$$\mathrm{REVAMT = Orig.\,REVAMT \times 1.06^{\frac{SWPDATE-TRXDATE}{365}}}$$

$$\mathrm{Org.\,REVAMT(t)} = \begin{cases} \mathrm{max(CURAMT(t) - TOTWD(t), 0)\ , t = 1} \\ \mathrm{max(CURAMT(t) - RMNGWD(t-1), 0)\ , t > 1} \end{cases}$$

$$\mathrm{RMNGWD(t)} = \begin{cases} \mathrm{TOTWD(t) - CURAMT(t), t = 1} \\ \mathrm{RMNGWD(t-1) - CURAMT(t), t > 1} \end{cases}$$

$$\mathrm{CURAMT = INITPREM \times ACCUM\_FCTOR}$$

ACCUM_FCTOR = lookup (SWPRATES at SWPDATE)

11) AV/REVAMT

Please see bullet point 10) for REVAMT

12) WD from GMV/GUAR VALUE WD

$$\mathrm{GUAR\ VALUE\ WD\ =\ GMINAMT - REVGMV}$$

$$\text{GMINAMT} = \text{INITPREM} \times 0.9 \times (1 + \text{ACCRATE})^{\frac{\text{VALDATE}-\text{TRXDT}^{2}}{365}}$$

$$\text{ACCRATE} = 0.03 \text{ for all policies}$$

$$\text{REVGMV(t)} = \begin{cases} \text{GMINAMT(t)} - \text{TOTWD(t)}, t = 1 \\ \text{GMINAMT(t)} - \text{RMNGWD1}(t-1), t > 1 \end{cases}$$

$$\text{RMNGWD1(t)} = \begin{cases} \text{TOTWD(t)} - \text{GMINAMT(t)}, t = 1 \\ \text{RMNGWD1}(t-1) - \text{GMINAMT(t)}, t > 1 \end{cases}$$

13) GMV/REVGMV

Please see bullet point 12)

14) CSV

$$\text{CSV} = \max\left(\text{REVAMT}, \text{REVGMV} \times 0.1 + \text{REVGMV} \times 0.9 \times \left(1 - \text{SC(DUR)}\right)\right)$$

$$\text{DUR} = \text{VAL YEAR} - \text{ISSUE YEAR} + (\text{VAL MONTH} \times 100 + \text{VAL DAY}) \geq (\text{ISSUE MONTH} \times 100 + \text{ISSUE DAY})$$

$$\text{SC} = [0.09, 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, 0.03, 0.02, 0.01, 0, 0, 0 \ldots 0], \text{can be extended with } 0$$

If DUR =3, then SC(DUR)=0.08, the 3rd item of SC

15) Stat Rsv

$$\text{Stat Rsv} = \max(\text{CSV}, \text{STAT\_CARVM})$$

[2] APL does not take leap year into consideration. So, in date difference calculation, we assume there are 365 days for all years. In the excel model, we keep the same calculation as APL for testing purpose. While in the future, Voya may adjust the date difference to fit reality.

$$\text{STAT\_CARVM} = \max_i\left(\text{PVDB}(i)^3 + \text{PVCSV}(i)\right)$$

$$\text{PVCSV}(i) = \begin{cases} 0, i = 1 \\ \max\left(\text{PROJGMV}(i), \text{PROJCSV}(i)\right) \times \text{PX}(i) \div (1 + \text{STATRT})^{\text{RMNGTERM}+20-\text{DUR}}, 1 < i \leq 22 - \text{DUR} \\ 0, i > 22 - \text{DUR} \end{cases},$$

$$\text{PVDB}(i) = \begin{cases} \sum_{j=1}^{i} \max\left(\text{PROJGMV}(j), \text{PROJCSV}(j)\right) \times \text{DEFQX}(j) \div (1 + \text{STATRT})^{\text{RMNGTERM}+j-2}, \text{if } i \leq 22 - \text{DUR} \\ 0, i > 22 - \text{DUR} \end{cases}$$

$$\text{PROJGMV}(i) = \begin{cases} \text{REVPREM} \times 0.9 \times 1.03^{\text{PREMDUR}+i-2}, i \leq 22 - \text{DUR} \\ 0, i > 22 - \text{DUR} \end{cases}$$

$$\text{PROJCSV}(i) = \begin{cases} \text{PROJAV}(i) \times \left(1 - (1 - \text{FWDPCT}) \times \text{SC}(\text{DUR})\right), \text{if DUR} = 1 \\ \text{PROJAV}(i) \times \left(1 - (1 - \text{FWDPCT}) \times \text{SC}(\text{DUR} - 1)\right), \text{if DUR} > 1 \end{cases}$$

FWDPCT=0.1, for all policies

$$\text{PROJAV}(i) = \begin{cases} \text{REVAMT}, i = 1 \\ (\text{REVAMT} + \text{NETMV\_S}) \times (1 + \text{ACCRATE})^{i-2}, 1 < i \leq 22 - \text{DUR} \\ 0, i > 22 - \text{DUR} \end{cases}$$

$$\text{NETMV\_S} = \text{NETMV} \times (1 + \text{STATRT})^{\text{RMNGTERM}}$$

$$\text{STATRT} = \text{lookup}(\text{STATRTS at ISS YEAR})$$

$$\text{PREMDUR} = \text{ISSYEAR} - \text{TRXYEAR} + (\text{ISS MONTH}) >= \text{TRX MONTH})$$

[3] t: index of each record (or row) for each policy, t =1 means the first and earliest transaction for each policy. i: the i th column for each variable

$QX = q_x = \text{lookup}\big(\text{MORTTBLS at the ceiling(ATTDAGE)}\big), \text{QX is a 1 by 15 array for each transaction}$

$\text{if issyear} < 2000, \text{lookup table IAM83, else lookup A2000, for female lookup column 3, male lookup column 2}$

$$ATTDAGE = ISSAGE + \frac{VAL\ DATE - ISSDATE}{365}$$

$$PX(i) = \prod_{j=x}^{i}\big(1 - QX(i)\big) = {}_{i}p_x = \prod_{j=x}^{i}(1 - q_j)$$

$$DEFQX(i) = PX(i-1) \times QX(i) = {}_{i-1|}q_x = {}_{i-1}p_x \times q_i$$

16) Option MV/NETMV

$$NETMV = \frac{CALLMV\_NET \times PART \times REVAMT}{SWPINX}$$

$$CALLMV\_NET = CALLMV\_P - CALLMV\_S$$

$$CALLMV\_P = VALIDX \times e^{-DIVRATE \times RMNGTERM} \times N\_D1P - SWPINX \times e^{-RFRATE \times RMNGTERM} \times N\_D2P$$

$$N\_D1P = \text{normcdf}(D1P)$$ [4]

$$N\_D2P = \text{normcdf}(D2P)$$

$$D1P = \frac{1}{VAL\_BID \times RMNGTERM}\left(\ln\left(\frac{VALIDX}{SI\_PUR}\right) + \left(RFRATE - DIVRATE + \frac{{VAL_{BID}}^2}{2}\right) \times (RMNGTERM)\right)$$

[4] In APL, the cumulative distribution function of the standard normal distribution is estimated by some polynomial approximation method. However, in Excel, it can be avoided

$$D2P = \frac{1}{VAL\_BID \times RMNGTERM}\left(\ln\left(\frac{VALIDX}{SI\_PUR}\right) + \left(RFRATE - DIVRATE - \frac{VAL_{BID}{}^{2}}{2}\right) \times (RMNGTERM)\right)$$

$$VAL\_BID = 0.01 \times lookup(VALTBL\ at\ LKUPDT)$$

$$CALLMV\_S = VALIDX \times e^{-DIVRATE \times RMNGTERM} \times N\_D1S - SI\_SAL \times e^{-RFRATE \times RMNGTERM} \times N\_D2S$$

$$N\_D1S = normcdf(D1S)$$

$$N\_D2S = normcdf(D2S)$$

$$D1S = \frac{1}{VAL\_ASK \times RMNGTERM}\left(\ln\left(\frac{VALIDX}{SI\_SAL}\right) + \left(RFRATE - DIVRATE + \frac{VAL\_ASK^{2}}{2}\right) \times (RMNGTERM)\right)$$

$$D2S = \frac{1}{VAL\_ASK \times RMNGTERM}\left(\ln\left(\frac{VALIDX}{SI\_SAL}\right) + \left(RFRATE - DIVRATE - \frac{VAL\_ASK^{2}}{2}\right) \times (RMNGTERM)\right)$$

$$VAL\_ASK = 0.01 \times lookup(VALTBL\ at\ LKUPDT)$$

DIVRATE = 0.0127507745 for all policies

RFRATE = lookup (RFRATES at the RMNGTERM)

$$RMNGTERM = max(1 - \frac{VALDATE - LKUPDT + 1}{365}, 0.0001)$$

$$LKUPDT = MRSWPYR\ \&\ TRX\ MONTH\ \&\ SWPDAY$$

$$MRSWPYR = VAL\ YEAR - (TRXMONTH > VAL\ MONTH)$$

$$SWPINX/SI\_PUR = lookup(S\&P500\ Index\ at\ the\ LKUPDT)$$

$$\text{SI\_SAL} = \left(1 + \frac{\text{CAP}}{\text{PART}}\right) \times \text{SWPINX}, \text{where CAP} = \text{lookup(PARTCAPS)}, \text{PART} = \text{lookup(PARTCAPS)}$$

17) GAV

$$\text{GAV} = (1 - \text{OPTCOST}) \times \text{REVAMT} \times (1 + \text{GAAPRT})^{\frac{\text{VALDATE}-\text{LKUPDT}}{365}}, \text{where OPTCOST} = 0.04 \text{ for all policies}$$

$$\text{GAAPRT} = \begin{cases} \frac{1}{1 - 0.04} - 1, \text{if REVAMT} \neq 0 \\ 0, \text{Otherwise} \end{cases}$$

18) ING GAAP Rsv

$$\text{INGGAAP} = \text{GAV} + \text{NETMV}$$

19) Tax Rsv

$$\text{Tax Rsv} = \min(\text{Stat Rsv}, \text{TAX\_CARVM})$$

$$\text{TAX\_CARVM} = \max_{\text{i}}(\text{PVDB(i)} + \text{PVCSV(i)})$$

$$\text{PVCSV(i)} = \begin{cases} 0, \text{i} = 1 \\ \max\left(\text{PROJGMV(i)}, \text{PROJCSV(i)}\right) \times \text{PX(i)} \div (1 + \text{TAXRT})^{\text{RMNGTERM}+20-\text{DUR}}, 1 < \text{i} \leq 22 - \text{DUR} \\ 0, \text{i} > 22 - \text{DUR} \end{cases},$$

$$\text{PVDB(i)} = \begin{cases} \sum_{\text{j}=1}^{\text{i}} \max\left(\text{PROJGMV(j)}, \text{PROJCSV(j)}\right) \times \text{DEFQX(j)} \div (1 + \text{TAXRT})^{\text{RMNGTERM}+\text{j}-2}, \text{if i} \leq 22 - \text{DUR} \\ 0, \text{i} > 22 - \text{DUR} \end{cases}$$

$$\text{PROJGMV(i)} = \begin{cases} \text{REVPREM} \times 0.9 \times 1.03^{\text{PREMDUR}+\text{i}-2}, \text{i} \leq 22 - \text{DUR} \\ 0, \text{i} > 22 - \text{DUR} \end{cases}$$

$$\text{PROJCSV(i)} = \begin{cases} \text{PROJAV(i)} \times \big(1 - (1 - \text{FWDPCT}) \times \text{SC(DUR)}\big), \text{if DUR} = 1 \\ \text{PROJAV(i)} \times \big(1 - (1 - \text{FWDPCT}) \times \text{SC(DUR} - 1)\big), \text{if DUR} > 1 \end{cases}$$

FWDPCT=0.1, for all policies

$$\text{PROJAV(i)} = \begin{cases} \text{REVAMT}, \text{i} = 1 \\ (\text{REVAMT} + \text{NETMV\_T}) \times (1 + \text{ACCRATE})^{\text{i}-2}, 1 < \text{i} \leq 22 - \text{DUR} \\ 0, \text{i} > 22 - \text{DUR} \end{cases}$$

$$\text{NETMV\_T} = \text{NETMV} \times (1 + \text{TAXRT})^{\text{RMNGTERM}}$$

$$\text{TAXRT} = \text{lookup(TAXRTS at ISS YEAR)}$$

$$\text{PREMDUR} = \text{ISSYEAR} - \text{TRXYEAR} + (\text{ISS MONTH}) >= \text{TRX MONTH})$$

$$\text{QX} = \text{q}_\text{x} = \text{lookup}\big(\text{MORTTBLS at ceiling(ATTDAGE)}\big), \qquad \text{QX is a 1 by 15 array for each transaction}$$

$$\text{if issyear} < 2000, \text{lookup table IAM83, else lookup A2000, for female lookup column 3, male lookup column 2}$$

$$\text{ATTDAGE} = \text{ISSAGE} + \frac{\text{VALDATE} - \text{ISSDATE}}{365}$$

$$\text{PX(i)} = \prod_{\text{j=x}}^{\text{i}} (1 - \text{QX(i)}) = {}_{\text{i}}\text{p}_\text{x} = \prod_{\text{j=x}}^{\text{i}} (1 - \text{q}_\text{j})$$

$$\text{DEFQX(i)} = \text{PX(i} - 1) \times \text{QX(i)} = {}_{\text{i}-1|}\text{q}_\text{x} = {}_{\text{i}-1}\text{p}_\text{x} \times \text{q}_\text{i}$$

20) At Iss Emb Opt/EMBOPT

$$\text{EMBOPT}^{5} = \sum_{i=1}^{20+\text{DROP2}} \text{EO\_CF}(i) \times \text{DISC}(i)$$

$$\text{DISC}(i) = \frac{1}{\left(1+\text{RFR}(i)\right)^{i}}$$

$$\text{RFR} = 0.01 \times \text{lookup}(\text{SPOTRTS at max}(\text{Transaction Date}, 09/01/2009))$$

$$\text{DROP2} = \text{TDUR} - \text{DUR} = -\text{DROP1}$$

$$\text{TDUR} = \text{Ceiling}(\text{TYRS})$$

$$\text{TYRS} = \frac{\text{VALDT} - \text{TRXDT}}{365}$$

$$\text{DUR} = \text{Ceiling}(\text{YRS})$$

$$\text{YRS} = \frac{\text{VALDT} - \text{ISSDT}}{365}$$

$$\text{EO\_CF} = \text{EO} \times (\text{BOY} - \text{EOY})$$

$$\text{EO} = \max\left(0, \text{ASSUMD\_AV} - \max(\text{INITPREM} - \text{WDAMT}, \text{MGV})\right)$$

$$\text{ASSUMD\_AV}(i) = \text{round}^{6}\left((\text{INITPREM} - \text{WDAMT}) \times \prod_{j=1}^{i}\left(1 + \text{AFG}(j)\right), 2\right)$$

$$\text{AFG}(i) = \text{BUD} \times \left(1 + \text{FRT}(i)\right), \text{where BUD} = 0.04 \text{ for all policies}$$

[5] t represents the row number or the t th transaction for each policy, i represents column number of the variable

[6] In APL the round (10.5, 2) represents $10^{-2} \times \text{floor}(0.5 + 10.5 \times 10^{2}) = 10.5$. In excel, we use round function to replace it

$$FRT(i) = \begin{cases} round(RFR(1), 6), i = 1 \\ round\left(\frac{(1+RFR(i))^{i}}{(1+RFR(i-1))^{i-1}} - 1, 6\right), i > 1 \end{cases}$$

$$MGV(i) = round\left(min(REVAMT, INITPREM - WDAMT) \times MGCVPCT \times (1 + GRT)^{i}, 2\right)$$

$$, where\ MGCVPCT = 0.9, GRT = 0.03$$

$$BOY(i) = \begin{cases} 1, \ i = 1 \\ BOY(i-1) \times \left(1 - TRT(DROP1 + i - 1)\right), i > 1 \end{cases}$$

TRT = [0.01, 0.01, 0.01, 0.01, 0.01, 0.02, 0.03, 0.04, 0.05, 0.05, 0.1, 0.1, 0.1, 0.1, 1, 1, … ,1]

$$EOY(i) = \begin{cases} 1 - TRT(DROP1 + 1), \ i = 1 \\ EOY(i-1) \times \left(1 - TRT(DROP1 + i)\right), i > 1 \end{cases}$$

21) At Iss Host Inst/HOSTINST

$$HOSTINST = INITPREM - WDAMT - EMBOPT$$

22) At Iss Tot Rsv

At Iss Tot Rsv=EMB_OPT+HOSTINST

23) Imp Int Rate/IMPINT

$$IMPINT = \begin{cases} 0, HOSTINST = 0 \\ \left(\frac{MGV(20 + DROP2)}{HOSTINST}\right)^{\frac{1}{KEEP}} - 1, \quad HOSTINST \neq 0 \end{cases}$$

$$KEEP = MAX(20 - DROP1,1)$$

24) Val Emb Opt/VEMBOPT

$$\text{VEMBOPT} = \sum_{\text{i}=1}^{20} \text{EO\_CF(i)} \times \text{DISC}(i)$$

$$\text{DISC(i)} = \frac{1}{\left(1 + \text{RFR(i)}\right)^{\text{i+floor(TYRS)}-\text{TYRS}}}$$

$$\text{RFR(i)} = \begin{cases} \text{Orig. RFR(floor(YTRS)} + \text{i), i} \leq \text{NUM} + 1 \\ 0, \text{i} > \text{NUM} + 1 \end{cases}$$

$$\text{Orig. RFR} = 0.01 \times \text{lookup(SPOTRTS at Valuation Date), where the size is t} \times 22$$

$$\text{NUM} = \text{KEEP} - \text{floor(TYRS)}$$

$$\text{EO\_CF} = \text{EO} \times (\text{BOY} - \text{EOY})$$

$$\text{EO(i)} = \max\left(0, \text{ASSUMD\_AV(i} + 1) - \max\left(\text{INITPREM} - \text{WDAMT, UGV(i} + 1)\right)\right)$$

$$\text{ASSUMD\_AV(i)} = \begin{cases} \text{REVAMT, i} = 1 \\ \text{ASSUMD\_AV(i} - 1) \times \left(1 + \text{AFG(i} - 1)\right), 1 < \text{i} \leq \text{NUM} + 1 \\ 0, \quad \text{i} > \text{NUM} + 1 \end{cases}$$

$$\text{AFG(i)} = \begin{cases} \text{BRT(i)} \times \left(1 + \text{FRT(i)}\right)^{\text{floor(TYRS)}+1-\text{YTRS}}, \text{i} = 1 \\ \text{BRT(i)} \times \left(1 + \text{FRT(i)}\right), \text{i} > 1 \end{cases}$$

$$\text{BRT(i)} = \begin{cases} \frac{\text{NETMV}}{\text{REVAMT}}, \text{i} = 1 \\ \text{BUD} = 0.04, 1 < \text{i} \leq \text{NUM} \\ 0, \text{i} > \text{NUM} \end{cases}$$

$$\mathrm{FRT(i)} = \begin{cases} \mathrm{RFR(i)}, \mathrm{i} \le 2 \\ \mathrm{round}\left(\frac{(1+\mathrm{RFR(i)})^{\mathrm{i}-1}}{(1+\mathrm{RFR(i-1)})^{\mathrm{i}-2}} - 1, 6\right), 2 < \mathrm{i} \le \mathrm{NUM} + 1 \\ 0, \mathrm{i} > \mathrm{NUM} + 1 \end{cases}$$

$$\mathrm{UGV(i)} = \begin{cases} \mathrm{REVGMV}, \mathrm{i} = 1 \\ \mathrm{MGV}\big(\mathrm{i} - 1 + \mathrm{floor(TYRS)}\big), 1 < \mathrm{i} \le \mathrm{NUM} + 1 \\ 0, \mathrm{i} > \mathrm{NUM} + 1 \end{cases}$$

$$\mathrm{BOY(i)} = \begin{cases} 1,\ \mathrm{i} = 1 \\ \mathrm{BOY(t, i-1)} \times \Big(1 - \mathrm{TRT}\big(\mathrm{DROP1} + \mathrm{i} - 1 + \mathrm{floor(TYRS)}\big)\Big), 1 < \mathrm{i} \le \mathrm{NUM} \\ 0, \mathrm{i} > \mathrm{NUM} \end{cases}$$

$$\mathrm{EOY(i)} = \begin{cases} \Big(1 - \mathrm{TRT}\big(\mathrm{DROP1} + 1 + \mathrm{floor(TYRS)}\big)\Big), \mathrm{i} = 1 \\ \mathrm{EOY(t, i-1)} \times \Big(1 - \mathrm{TRT}\big(\mathrm{DROP1} + \mathrm{i} + \mathrm{floor(TYRS)}\big)\Big), 1 < \mathrm{i} \le \mathrm{NUM} \\ 0, \mathrm{i} > \mathrm{NUM} \end{cases}$$

25) Val Host Inst/VHOST

$$\mathrm{VHOST} = \mathrm{HOSTINST} \times (1 + \mathrm{IMPINT})^{\mathrm{TYRS}}$$

26) Val Tot Rsv

$$\mathrm{Val\ Tot\ Rsv} = \mathrm{VEMBOPT} + \mathrm{VHOST}$$

27) FAS157 Emb Opt/VFEMBOPT

$$\mathrm{VFEMBOPT} = \sum_{\mathrm{i}=1}^{20} \mathrm{EO\_CF(i)} \times \mathrm{DISC157(i)}$$

$$\text{DISC157(i)} = \frac{1}{\left(1 + \text{RFR157(i)}\right)^{\text{i+floor(TYRS)}-\text{TYRS}}}$$

$$\text{RFR157(i)} = \text{RFR(i)} + \text{lookup(SPREADS at (i + NUM) row and valuation date)}$$

4. Appendix

Variables	**Category**[7]	**Column**	**Explanation**	**Usage**[8]
ACCUM_FCTOR	I	H	Accumulation factor	A
AFG	M	MZ-NS for At Iss Emb Opt YH-ZA for Val Emb Opt	Adjusted forward rate	G
ASSMD_AV	M	NT-OM for At Iss Emb Opt ZB-ZV for Val Iss Emb Opt	Assumed account value	G
At Iss Tot Rsv	O	SL	Total IFRS Reserve as of issue date	G
ATTDAGE	M	Z	Attained age	S, T
AV WD	O	U	Actual total withdraw amount from account value as of transaction date	A
BOY	M	PH-QA for At Iss Emb Opt AAQ-ABJ for Val Emb Opt	Beginning of the year rate	G
BRT	M	XN-YG	Bud rate	G

[7] I represents input variable, O for output variable, M for intermediate variable
[8] S represents statutory reserve, T represents tax reserve, G represents GAAP reserve, A represents all three reserves

Variables	Category[7]	Column	Explanation	Usage[8]
CALLMV_NET	M	AZ	Net market value of the combination of call options	A
CALLMV_P	M	AT	Market value of the call option P	A
CALLMV_S	M	AY	Market value of the call option S	A
CSV	O	T	Cash value as of valuation date	S, T
CURAMT	M	I	Current amount, accumulated deposit as of valuation date	A
D1P	M	AP	d1 of option P	A
D1S	M	AU	d1 of option S	A
D2P	M	AR	d2 of option P	A
D2S	M	AW	d2 of option S	A
DEFQX	M	CK-CY	Deferred mortality rate	A
DISC	M	RP-SI for At Iss Emb Opt ACY-ADR for Val Emb Opt	Discount factor, discounted risk-free rate	G

Variables	Category[7]	Column	Explanation	Usage[8]
DISC157	M	ADS-AEL for FAS157 Emb Opt	Discount factor for FAS157	G
DROP1	M	KM	Duration from each transaction to issue	G
DROP2	M	KN	Duration from each transaction to issue	G
DUR	M	S for STAT/TAX KJ for GAAP	Duration from issue to valuation date	A
EMBOPT	O	SJ	Embedded option reserve as of issue date	G
EO	M	ON-PG for At Iss Emb Opt ZW-AAP for Val Emb Opt	Embedded option	G
EO_CF	M	QV-RO for At Iss Emb Opt ACE-ACX for Val Emb Opt	Liability cash flow	G
EOY	M	QB-QU for At Iss Emb Opt ABK-ACD for Val Emb Opt	End of the year rate	G

Variables	**Category**[7]	**Column**	**Explanation**	**Usage**[8]
FRT	M	LL-ME for At Iss Emb Opt UH-VB for Val Emb Opt	Forward rate	G
FRT157	M	VX-WR for FAS157 Emb Opt	Forward rate for FAS157	G
GAAPRT	M	BA	GAAP rate	G
GAV	O	BB	Guaranteed account value	G
GMINAMT	M	J	Guaranteed minimum amount	S, T
GUAR VALUE WD	O	V	Withdraw from guaranteed minimum value	S, T
HOSTINST	O	SK	Host instrument at issue	G
IMPINT	O	SM	Implied interest rate	G
INGGAAP	O	BD	GAAP reserve based on each transaction/deposit/premium, not the Total US GAAP to be reported	G
KEEP	M	KO	Duration from transaction to termination	G
LKUPDT	M	AD-AE	Lookup date for sweep rate	S, T

Variables	**Category**[7]	**Column**	**Explanation**	**Usage**[8]
MGV	M	MF-MY	Minimum guaranteed value for At Iss Emb Opt	G
N_D1P	M	AQ	N(d1) of option P	A
N_D1S	M	AV	N(d1) of option S	A
N_D2P	M	AS	N(d2) of option P	A
N_D2S	M	AX	N(d2) of option S	A
NETMV	O	BC	Market value of the option Net market value of the guaranteed minimum return	A
NETMV_S	M	BE	Statutory market value of the option Accumulated net market value using stat rate	S
NETMV_T	M	BF	Tax market value of the option Accumulated net market value using tax rate	S
NUM	M	SN	Projection duration for GAAP reserve	G
Org.REVAMT	M	L	Original revised account value	S, T
Orig.RFR	I	SQ-TL for Val Emb Opt	Spot rate, risk free rate	G
PART	I	AH	Participate rate	G

Variables	Category[7]	Column	Explanation	Usage[8]
PREMDUR	M	Y	Duration from issue to transaction	S
PROJAV	M	CZ-DT for STAT GQ-HK for TAX	Projected account value using CARVM	S, T
PROJCSV	M	EP-FJ for STAT IG-JA for TAX	Projected cash surrender value using CARVM	S, T
PROJGMV	M	DU-EO for STAT H-IF for TAX	Projected guaranteed minimum amount using CARVM	S, T
PVCSV	M	FZ-GN for STAT JQ-KE for TAX	Present value of cash surrender value using CARVM	S, T
PVDB	M	FK-FY for STAT JB-JP for TAX	Present value of death benefit using CARVM	S, T
PX	M	BV-CJ	Survival rate	S, T
QX	M	BG-BU	Mortality rate	S, T
REVAMT	O	R	Revised account value, net account value after withdraw as of valuation date	A

Variables	Category[7]	Column	Explanation	Usage[8]
REVGMV	O	M	Revised guaranteed minimum value, net guaranteed minimum value after withdraw as of valuation date	A
REVPREM	O	K	Revised premium, net premium after withdraw	A
RFATE	I	AO	Risk free rate	G
RFR	I	KP-LK for At Iss Emb Opt TM-UG for Val Emb Opt	Spot rate	G
RFR157	M	VC-VW for FAS157 Emb Opt	Spot rate for FAS157 Emb Opt	G
RMNGTERM	M	AM	Remaining term	A
RMNGWD	M	O	Remaining withdraw	A
RMNGWD0	M	N	Remaining withdraw	A
RMNGWD1	M	P	Remaining withdraw	A
SI_PUR/SWPINX	M	AI	Strike price for the call option P	A
SI_SAL	M	AK	Strike price for the call option S	A
STAT RSV	O	GP	Final statutory reserve	S

Variables	Category[7]	Column	Explanation	Usage[8]
STAT_CARVM	M	GO	Statutory reserve using CARVM	S
STATRT	I	W	Stat rate	S
TAX RSV	O	KG	Final tax reserve	T
TAX_CARVM	O	KF	Tax reserve using CARVM	T
TAXRT	I	X	Tax rate	T
TDUR	M	KK	Duration from transaction to valuation date	G
TYRS	M	KJ	Duration from transaction to valuation date	G
UGV	M	WS-XM	Minimum guaranteed value for Val Emb Opt	G
Val Tot Rsv	O	AEP	Total IFRS Reserve as of valuation date	G
VAL_ASK	M	AM	Ask volatility	A
VAL_BID	M	AL	Bid volatility	A
VALIDX	I	AJ	Index on the valuation date (spot price)	A
Vantage AV	O	AEQ	System account value	A
VEMBOPT	O	AEM	Embedded option as of valuation date	G
VFEMBOPT	O	AEN	FAS157 embedded option	G

Variables	Category[7]	Column	Explanation	Usage[8]
VHOST	O	AEO	Host instrument at valuation date	G
WDAMT	O	Q	Withdraw amount, allocate total withdraw (TOTWD) using FIFO (First In First Out)	A
YRS	M	KH	Duration from issue to valuation date	G

Notice: Some variables do not have explanations since they are intermediate variables and difficult to explain the meaning. For these variables, please see the formulas in the excel model "*Voya EIA Reserve Calculation (final version 3).xlsx*".